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Deformation Analysis of Variable Thickness Rotating Disks Using an Improved Adomian Decomposition Technique

Servet Mert Kutsal* and Safa Bozkurt Coşkun†
Department of Civil Engineering, Kocaeli University
Umuttepe, Kocaeli 41380, Turkey
**servetmertt@gmail.com*
†*sb.coskun@kocaeli.edu.tr*

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In this study, axisymmetric elastic deformation analysis of rotating disks with variable thickness is conducted using an improved Adomian decomposition method (IADM). Variation of thickness is assumed as hyperbolic and different variations are employed for each case considered. Several analytical approximate solutions in different orders are obtained for radial stress, tangential stress, radial displacement and are compared with exact solutions. Results show that IADM can effectively be used in the deformation analysis of rotating variable thickness disks providing the solution as an analytical continuous function in the solution domain.

Keywords: Rotating disk; variable thickness; Adomian decomposition method; deformation; stress.

1. Introduction

In various disciplines of engineering, such as mechanical and structural engineering, rotating disks have a wide application area as the basic parts for machineries such as turbines, generators, flywheels, pumps, etc. Due to its critical role on design of these structures, rotating disks require an accurate analysis of displacement and stresses.

Rotating disk problems have been tackled by different researchers and they still attract great attention. Gamer performed elastic–perfectly plastic analysis of rotating solid disks with uniform thickness [Gamer, 1983] and later the study was extended for linearly strain hardening materials [Gamer, 1984a; 1984b; 1985]. Guven solved rotating solid and annular disk problems assuming variable thickness

*Corresponding author.

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[Guven, 1992, 1994, 1995]. Eraslan and Orcan [2002] and Orcan and Eraslan [2002] produced closed-form solutions for elastic–plastic analyses of variable thickness rotating disk problems. Eraslan and Argeso [2012] investigated effects of the thickness function on limit angular velocities using Von-Misses yield criterion. There exists also perturbation [You and Zhang, 1999] and numerical solutions [You *et al.*, 1997]. Elastic–plastic problem was also studied using some analytical approximation techniques [Hojjati and Jafari, 2008, 2009]. Argeso [2012] solved rotating disc problem with three parameter variation functions including variation both in thickness and material properties. Zafarmand and Hassani [2014] applied the graded finite element method for the analysis of two-dimensional functionally graded rotating annular and solid disks. Eraslan and Ciftci [2015] used an exponential thickness profile and solved rotating disk problem analytically and numerically. Nayebi *et al.* [2015] considered elastic–plastic behavior of a functionally grading rotating disk under cyclic-thermo mechanical loadings. Sahni and Sahni [2015] considered rotating functionally graded disks with variable thickness profile under external pressure. Zheng *et al.* [2016] studied functionally graded rotating disks with variable thickness and variable angular velocity. Dai and Dai [2016] used a semi-analytical approach for the thermo-elastic analysis of a functionally graded rotating hollow circular disk. [Kalali *et al.*, 2016a,b] presented an elastic–plastic stress solution for a functionally graded rotating disk problem. [Thawait *et al.*, 2017a,b] studied analysis of variable thickness rotating disks made of functionally graded materials. Jalali and Shahriari [2018] considered elastic deformation analysis of functionally graded rotating disk using finite difference method. Allam *et al.* [2018] presented semi-analytical solutions for stress analysis of functionally graded rotating annular disks with arbitrary thickness variations. Bagheri and Jahangiri [2019] used differential quadrature method for the analysis of in-plane vibration and critical speeds of functionally graded rotating disks.

Previous works revealed that the stresses and displacements occurring in variable thickness disks are lower than the uniform disks rotating at the same speed. This issue may provide possible design choices for more durable disks with less material. Hence, it is important to use accurate solution techniques. In this study, a modified Adomian decomposition method is used for solving the variable thickness rotating disk problem. The method was proposed specifically for singular two-point boundary value problems [Ebaid, 2011] and applied successfully to some fundamental problems in applied mathematics. The method is also suitable for the use in rotating disk problems in this sense such that analytical approximations for different variations in thickness function can be obtained.

2. Variable Thickness Rotating Disks

Equilibrium for rotating disks with thickness variation is considered here without any derivation. However, the reader may refer to Ugural and Fenster [2003] and

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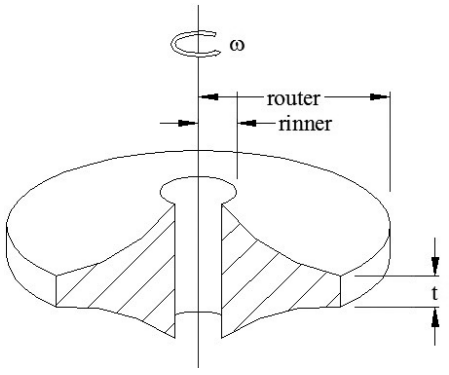


Fig. 1. Disk of hyperbolic profile.

Timoshenko and Goodier [1970] for detailed derivations and further information. The type of disk considered in this study is shown in Fig. 1.

Equation of equilibrium for rotating disks with varying thickness is given as follows:

$$\frac{d}{dr}(tr\sigma_r) - t\sigma_\theta + t\rho\omega^2 r^2 = 0. \tag{2.1}$$

In the equation, t is the variable thickness, r is the radius, σ_r is the radial stress, σ_θ is the tangential (circumferential) stress, ρ is the mass density and ω is the constant angular speed of the disk in rad/s. Equation (2.1), the equation of equilibrium, may be satisfied by a stress function of the following form [Ugural and Fenster, 2003]:

$$\phi = tr\sigma_r. \tag{2.2}$$

Inserting Eq. (2.2) into Eq. (2.1) produces a relation for tangential stress in the following form:

$$\frac{d\phi}{dr} + t\rho\omega^2 r^2 = t\sigma_\theta. \tag{2.3}$$

Strain–displacement relations given in Eq. (2.4) as follows:

$$\varepsilon_r = \frac{du}{dr} = \frac{1}{E}(\sigma_r - v\sigma_\theta) \quad \varepsilon_\theta = \frac{u}{r} = \frac{1}{E}(-v\sigma_r + \sigma_\theta)\gamma_{r\theta} = 0, \tag{2.4}$$

lead to the compatibility equation as follows:

$$\frac{du}{dr} - \varepsilon_r = \frac{d}{dr}(r\varepsilon_\theta) - \varepsilon_r = r\frac{d\varepsilon_\theta}{dr} + \varepsilon_\theta - \varepsilon_r = 0. \tag{2.5}$$

Inserting the strain–stress relations given in Eq. (2.4) into Eq. (2.5) in view of stress functions for radial and tangential stresses given in Eqs. (2.2) and (2.3) produce the following equation:

$$r^2\frac{d^2\phi}{dr^2} + \left(1 - \frac{r}{t}\frac{dt}{dr}\right)r\frac{d\phi}{dr} + \left(\nu\frac{r}{t}\frac{dt}{dr} - 1\right)\phi = -(3+\nu)\rho\omega^2 tr^3. \tag{2.6}$$

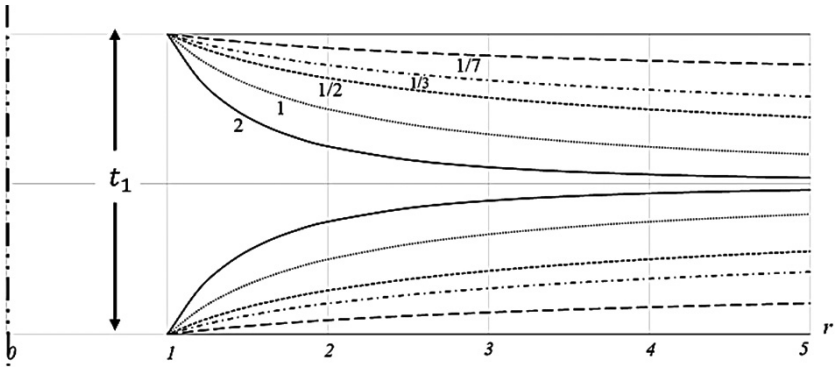


Fig. 2. Disk profiles for different values of s.

In Eq. (2.6), ν is Poisson ratio. A hyperbolic profile is assumed for the thickness in the following form [Ugural and Fenster, 2003]:

$$t = t_1 r^{-s}, \tag{2.7}$$

t_1 is the thickness at radius equal to unity. Figure 2 depicts various disk profiles for different values of s.

Inserting Eq. (2.7) into Eq. (2.6) gives

$$r^2 \frac{d^2\phi}{dr^2} + (1+s) r \frac{d\phi}{dr} - (1+\nu s)\phi = -(3+\nu)\rho\omega^2 t_1 r^{3-s}. \tag{2.8}$$

After solving Eq. (2.8) for stress function ϕ, by putting that solution into Eq. (2.2) radial stress can be derived and by putting that solution into Eq. (2.3) tangential stress can be derived. Finally, the displacement can be produced from the stress–strain relations given in Eq. (2.4).

2.1. *Exact solutions*

Exact solutions are obtained for different values of s, from 0.25 to 2.00 with 0.25 increments. Hence, eight different variations are considered. Mathematica software is used for every mathematical operation in the study. Besides, the reader may refer to Timoshenko and Goodier [1970, pp. 82–83] and to Ugural and Fenster [2003, pp. 331–332] for the general form of the exact solution.

For the cases considered in this study, exact solutions are as follows:

For $s = 0.25$:

$$\sigma_r = \frac{-6.8391 * 10^{-5} + 0.0316 r^{2.08866} - 0.4599 r^{3.91933}}{r^{1.91933}}. \tag{2.9}$$

For $s = 0.5$:

$$\sigma_r = \frac{-6.8997 * 10^{-5} + 0.0543 r^{2.20227} - 0.5196 r^{3.85114}}{r^{1.85114}}. \tag{2.10}$$

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For $s = 0.75$:

$$\sigma_r = \frac{-6.6832 * 10^{-5} + 0.08101r^{2.3372} - 0.5972r^{3.7936}}{r^{1.7936}}. \tag{2.11}$$

For $s = 1.00$:

$$\sigma_r = \frac{-6.2724 * 10^{-5} + 0.1252r^{2.48998} - 0.7019r^{3.74499}}{r^{1.74499}}. \tag{2.12}$$

For $s = 1.25$:

$$\sigma_r = \frac{-5.7502 * 10^{-5} + 0.2019r^{2.65754} - 0.8516r^{3.70377}}{r^{1.70377}}. \tag{2.13}$$

For $s = 1.50$:

$$\sigma_r = \frac{-5.1845 * 10^{-5} + 0.3443r^{2.83725} - 1.0819r^{3.66863}}{r^{1.66863}}. \tag{2.14}$$

For $s = 1.75$:

$$\sigma_r = \frac{-4.6231 * 10^{-5} + 0.6384r^{3.02696} - 1.4831r^{3.63848}}{r^{1.63848}}. \tag{2.15}$$

For $s = 2.00$:

$$\sigma_r = \frac{-4.9604 * 10^{-5} + 1.38098r^{3.2249} - 2.35714r^{3.61245}}{r^{1.61245}}. \tag{2.16}$$

$s = 0$ corresponds to constant thickness rotating disk case. This problem was previously examined by Coşkun and Kara [2018]. The case $s = 1$ was also previously solved by Mert Kutsal and Coşkun [2018].

In the following sections, first, the method used in this study will be outlined and then it will be applied to different cases for which the exact solutions are given above.

3. Improved Adomian Decomposition Method (IADM)

Many problems in applied mathematics end up with a two-point boundary value problem of the form

$$u''(x) + p(x)u'(x) + q(x)f(u(x)) = r(x), \tag{3.1}$$

subject to the boundary conditions $u(a) = \alpha$ and $u(b) = b$. In Eq. (3.1) $p(x)$, $q(x)$ and $r(x)$ are functions and a, b, α and β are finite constants. Ebaid [2011] proposed an operator based on the work by Lesnic [2001]. The use of this proposed operator with Adomian decomposition method [Adomian, 1994] generated an IADM for solving singular boundary value problems. By using IADM an analytical approximation for Eq. (2.8) can be obtained from following equation:

$$u(x) = u(a) + \frac{x-a}{b-a}[u(b) - u(a)] + L_{xx}^{-1}[r(x)] - L_{xx}^{-1}[p(x)u'(x)] - L_{xx}^{-1}[q(x)f(u(x))]. \tag{3.2}$$

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IADM uses Eq. (3.2) for the solution with the following inverse operator [Ebaid, 2011]:

$$L_{xx}^{-1}(\cdot) = \int_a^x dx' \int_c^{x'} (\cdot)dx'' - \frac{x-a}{b-a} \int_a^b dx' \int_c^{x'} (\cdot)dx'', \tag{3.3}$$

provided that $a \neq b$ and c is arbitrary. According to ADM dependent variable u and nonlinear term $f(u)$ may be decomposed as follows:

$$u = \sum_{n=0}^{\infty} u_n \quad f(u) = \sum_{n=0}^{\infty} A_n, \tag{3.4}$$

where A_n is Adomian's polynomial which can be calculated from the well-known formula as follows:

$$A_n = \frac{1}{n!} \left[\frac{d^n}{d\lambda^n} f \left(\sum_{i=0}^{\infty} \lambda^i u_i \right) \right]_{\lambda=0} \quad n \geq 0. \tag{3.5}$$

Substituting Eq. (3.3) into Eq. (3.2) in view of Eq. (3.4), the solution can be computed from the recurrence relations given as follows:

$$\begin{aligned} u_0(x) &= u(a) + \frac{x-a}{b-a}[u(b) - u(a)] + L_{xx}^{-1}[r(x)] \\ u_{n+1}(x) &= -L_{xx}^{-1}[p(x)u_n'(x) + q(x)A_n(x)] \quad n \geq 0, \end{aligned} \tag{3.6}$$

4. Case Studies

A rotating annular disk having parabolic profile with inner radius of 0.05 m and outer radius of 0.25 m is considered. The disk is subjected to centrifugal inertia force as can be introduced as body force in the equation of equilibrium. The boundary conditions for this problem are $(\sigma_r)_{r=\text{rinner}} = 0$ and $(\sigma_r)_{r=\text{router}} = 0$. The thickness at inner boundary is 0.075 m. Taking modulus of elasticity as 200 GPa, Poisson's ratio as 0.3 and $\rho\omega^2$ as unity. In the solution, the dimension used for E is Pa, for r is m, ρ is kg/m^3 and for ω is rad/s that lead to the dimension of displacement is m and the dimension of stress is Pa. Stresses and deformation are investigated using IADM.

In order to clear the application process, some intermediate results and the second order solution for the case of $s = 1.5$ is given as follows.

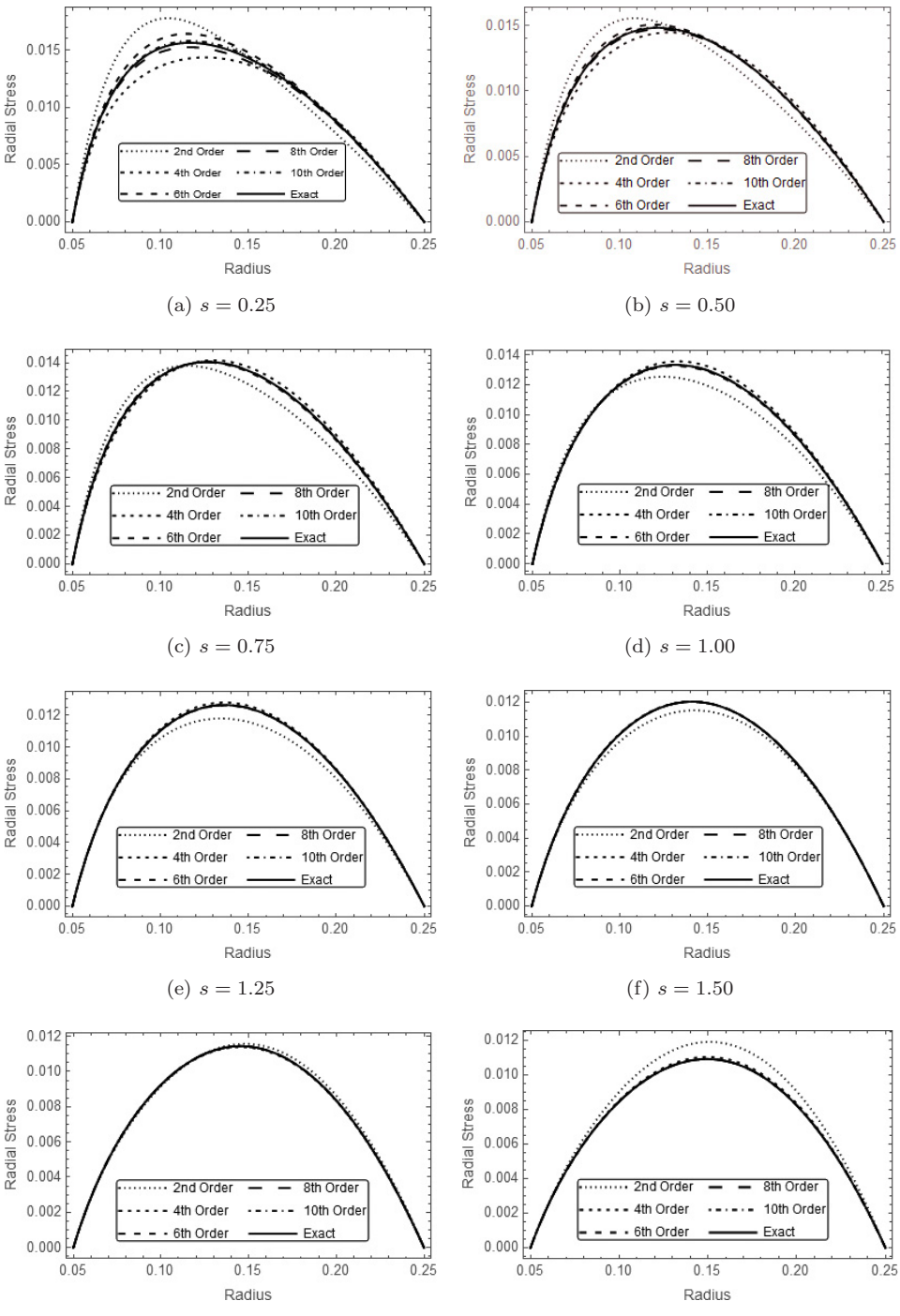
The use of Eq. (3.6) provides an initial approximation for radial stress as

$$\sigma_{r0} = -0.020625 + 0.495r - 1.65r^2, \tag{4.1}$$

where radius r varies between r_{inner} and r_{outer}. Inserting Eq. (4.1) into the second equation in Eq. (3.5) for $n \geq 0$, the next approximation term becomes

$$\begin{aligned} \sigma_{r1} = {} & 0.1699 - 0.36435141r + 0.86625r^2 \\ & + 0.04021875Ln(r) + 0.22275rLn(r). \end{aligned} \tag{4.2}$$

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(a) $s = 0.25$ (b) $s = 0.50$

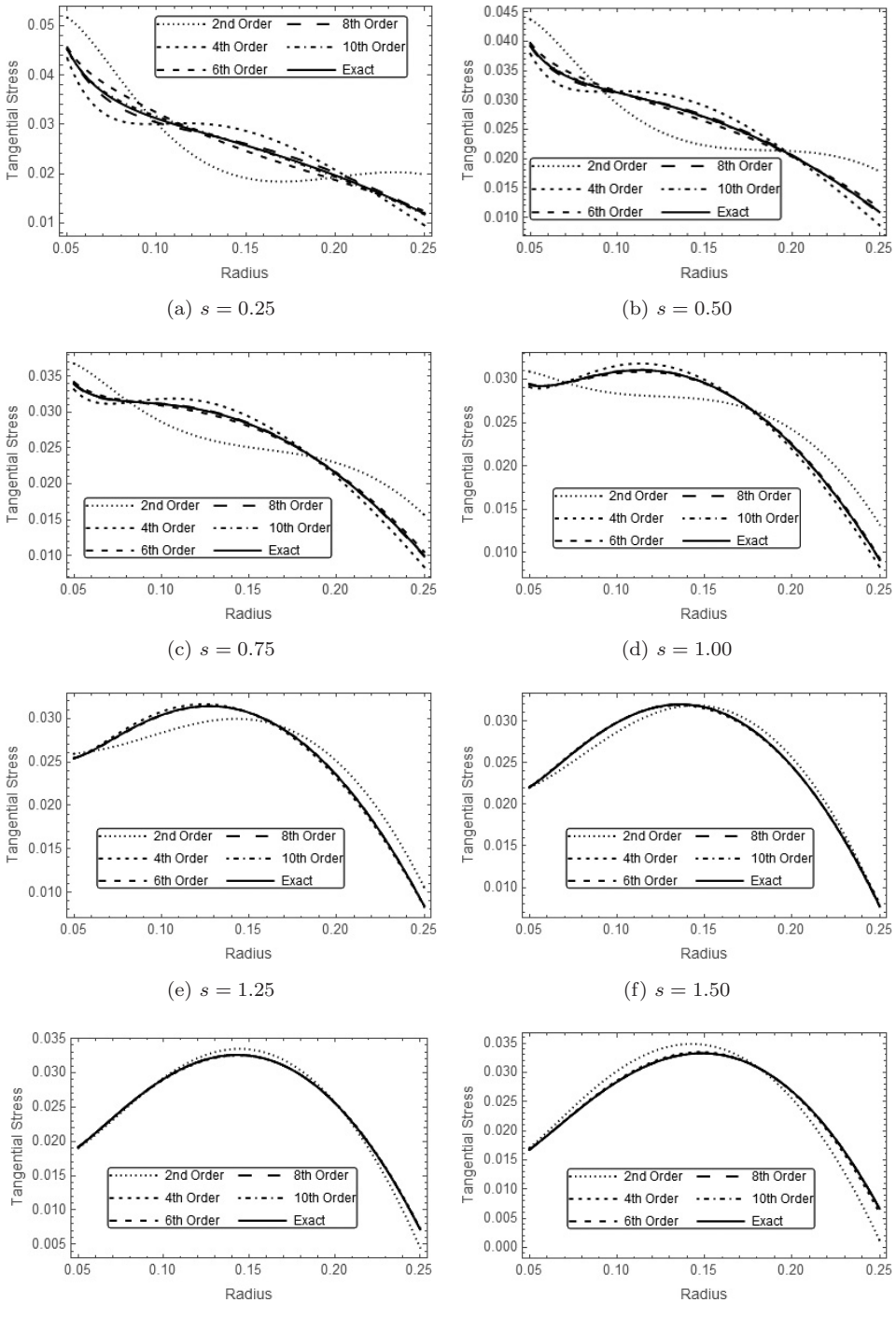
(c) $s = 0.75$ (d) $s = 1.00$

(e) $s = 1.25$ (f) $s = 1.50$

(g) $s = 1.75$ (h) $s = 2.00$

Fig. 3. Comparison of radial stresses for different values of s.

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(a) $s = 0.25$ (b) $s = 0.50$

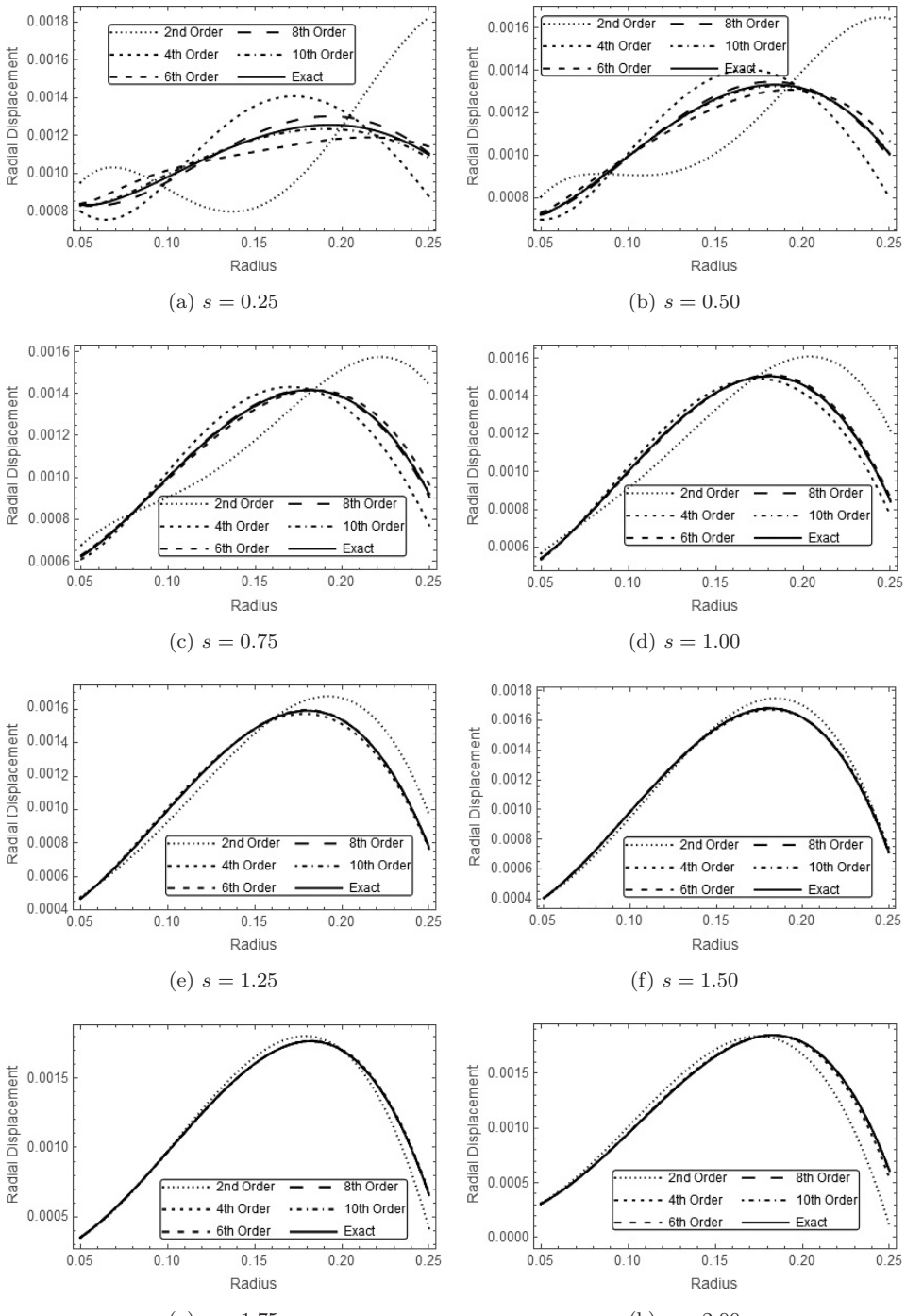
(c) $s = 0.75$ (d) $s = 1.00$

(e) $s = 1.25$ (f) $s = 1.50$

(g) $s = 1.75$ (h) $s = 2.00$

Fig. 4. Comparison of tangential stress diagrams for different values of s.

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(a) $s = 0.25$ (b) $s = 0.50$

(c) $s = 0.75$ (d) $s = 1.00$

(e) $s = 1.25$ (f) $s = 1.50$

(g) $s = 1.75$ (h) $s = 2.00$

Fig. 5. Comparison of radial displacement diagrams for different values of s.

Hence, a first-order solution for radial stress may be obtained as follows:

$$\sigma_r^{(1)} = \sigma_{r0} + \sigma_{r1} = 0.149375 - 0.85635141r - 0.78375r^2 + 0.04021875Ln(r) + 0.22275rLn(r). \quad (4.3)$$

The results for radial stress, tangential stress and radial displacement are compared graphically in Figs. 3, 4 and 5. The results suggest that fourth order solutions for radial and tangential stresses are in good agreement with the exact solutions for $s \geq 0.75$. Additional terms improve the accuracy to provide a perfect agreement. Variations for $s < 0.75$ require two more approximations for the same order of accuracy. Radial displacements require additional one or two terms. A tenth order solution for all stresses and radial deformation have perfect agreement with the exact solutions for all the variations considered in this study.

5. Conclusions

Stress and deformation analyses of a variable thickness rotating disk are considered with the use of an improved Adomian decomposition technique. The method was previously used in the solution of some singular differential equations in applied mathematics like Lane Emden equation.

Adomian decomposition method is an analytical approximation technique which was successfully used in the solution of linear/nonlinear deterministic/stochastic ordinary/partial differential equations.

Since the deformation problem of variable thickness rotating disk leads to a singular two-point boundary value problem, IADM has a possible application area in the problem presented in this study.

To this aim, a hyperbolic variation in thickness was assumed and a parametric study was conducted based on the variation in thickness of the disk. Solutions with different orders for radial stress, tangential stress and radial displacement were obtained for each case to investigate the order of solution sufficient to represent the solution adequately. Hence, second order, fourth order, sixth order, eighth order and tenth order solutions were obtained for each assumed thickness variation. Total number of cases is eight for all radial and tangential stresses and radial displacement that lead to total 120 solutions included in the study.

The results show that fourth to sixth order solutions are in good agreement with the analytical solutions for assumed variations with $s \geq 0.75$. While $s < 0.75$ two more approximations are required for the same order of accuracy. Eighth order solutions have perfect agreement with the analytical solutions for assumed variations with $s \geq 0.75$. For $s < 0.75$ tenth order solution provide prefect agreement.

Comparison of results given between Figs. 3 and 5 suggest that IADM can be successfully used in the deformation analysis of variable thickness rotating disks with a predefined mathematical expression for thickness variation in the interval where the problem is defined.

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