ICMASE 2025









2025 ABSTRACT BOOK

VI INTERNATIONAL CONFERENCE ON MATHEMATICS AND ITS APPLICATIONS IN SCIENCE AND ENGINEERING (ICMASE 2025)

15-17 July 2025

University of Plovdiv Paisii Hilendarski

Plovdiv/ Bulgaria

ABSTRACT BOOK

VI International Conference on Mathematics and Its Applications in Science and Engineering (ICMASE 2025)

University of Plovdiv Paisii Hilendarski,
Plovdiv / BULGARIA,
15-17 July 2025

Preface

This abstract booklet includes the abstracts of the papers that have been presented at VI International Conference on Mathematics and its Applications in Science and Engineering (ICMASE 2025) which was held in University of Plovdiv Paisii Hilendarski, Bulgaria between 15-17 July, 2025, via hybrid. The aim of this conference is to exchange ideas, discuss developments in mathematics, develop collaborations and interact with professionals and researchers from all over the world about some of the following interesting topics: Functional Analysis, Approximation Theory, Real Analysis, Complex Analysis, Harmonic and non-Harmonic Analysis, Applied Analysis, Numerical Analysis, Geometry, Topology and Algebra, Modern Methods in Summability and Approximation, Operator Theory, Fixed Point Theory and Applications, Sequence Spaces and Matrix Transformation, Modern Methods in Summability and Approximation, Spectral Theory and Diferantial Operators, Boundary Value Problems, Ordinary and Partial Differential Equations, Discontinuous Differential Equations, Convex Analysis and its Applications, Optimization and its Application, Mathematics Education, Applications on Variable Exponent Lebesgue Spaces, Applications on Differential Equations and Partial Differential Equations, Fourier Analysis, Wavelet and Harmonic Analysis Methods in Function Spaces, Applications on Computer Engineering, and Flow Dynamics. However, the talks are not restricted to these subjects.

Thanks to all committee members.

We wish everyone a fruitful conference and pleasant memories from ICMASE 2025.

Prof. Dr. Snezhana GOCHEVA-ILIEVA

Prof. Dr. Fatih YILMAZ

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VI International Conference on Mathematics and Its Applications in Science and Engineering (ICMASE 2025)

University of Plovdiv Paisii Hilendarski,
15-17 July 2025

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INVITED TALKS

Invited Speakers

- **Boyan ZLATANOV**, University of Plovdiv Paisii Hilendarski, (Bulgaria)
  Title: Study of Market Equilibrium in Oligopoly Markets Using Response Functions
- **Daniela RICHTARIKOVA**, Slovak University of Technology in Bratislava, (Slovakia)
  Title: On Pedagogical Aspects of Modern Tertiary Engineering Mathematics
- **Metin ORBAY**, Ankara Hacı Bayram Veli University, (Türkiye)
  Title: Optimal Title Length in Math?

Title: Study of Market Equilibrium in Oligopoly Markets Using Response Functions

Abstract: We present the theory of studying market equilibrium in oligopoly markets using a response function instead of maximizing the payoff functions of market participants. The considered technique is based on generalized coupled, tripled, and n-tuples of fixed points. The notion for a generalized type of the mentioned above fixed points naturally arises in the investigation of market equilibrium in an oligopoly market because the classical coupled and tripled are not suitable. We give the relationship between the two models, the one based on payoff maximization and the one using response functions. We get theoretical results that give sufficient conditions for the existence of market equilibrium. We obtain necessary and sufficient conditions under which, if the response function model is obtained by maximizing payoff, then the two models are equivalent. We illustrate the results obtained with markets of different types: duopoly markets and markets with three producers (tripodal markets); markets where competition is on just one good and markets where participants compete on many goods; markets that include the Cournot-Bertrand model, i.e., competition on quantities and prices simultaneously; and markets in which payoff functions are not differentiable and it is not naturally possible to apply payoff maximization techniques. We present models generated from real data and investigate the equilibrium and its stability using the developed theory of response functions.

Title: On Pedagogical Aspects of Modern Tertiary Engineering Mathematics

„When automatic control fails, the nuclear power plant operator must be capable to set all processes manually.“
From the speech by a nuclear power plant operator
to students, FME STU in Bratislava, April 2025

Abstract: Each era brings its own challenges. Only very recently, a new period in Digital era has been identified, the Fourth industrial revolution characterised by the robust commencement of ubiquitously mentioned artificial intelligence (AI), cloud services, and advanced robotics among others. The instant general availability of new technologies, specifically AI, appeals to a rapid response in education systems as well. Despite the fact that new, primarily systematic questions arise, such as how to grasp AI in particular, and how to work with it at schools, traditional problems of teaching and learning process are still relevant, especially with respect to new, up to date circumstances. Which mathematical skills and what mathematical content will be valuable for future university engineering studies and which of them for following professional life? Will the standard competences in mathematics have to be reviewed? What advantages and what risks do new technologies bring to pedagogy? How to reach the minimal standard competences, and how to set up the ways for their improvement? Which pedagogical methods will stay efficient, and which of them will turn out to be outdated? What soft skills will have to be fostered at schools?

And what will keep engineering students to learn mathematics? And will new generation be willing to study engineering at all?

Title: Optimal Math Title Length?

Abstract: Article titles represent the first point of contact for editors, reviewers, and readers. In the context of the exponential growth in published articles, their importance has increased significantly. Therefore, researchers who aim to enhance the visibility of their work in international journals must give careful consideration to the title formulation process. In this study, we will discuss what the optimal title length should be in mathematics articles.

Keywords: Citation analysis, Research impact, Research visibility.

CONTRIBUTED TALKS

ICMASE 2025

ON GEOMETRIC FRANK MATRICES AND THEIR PROPERTIES

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ABSTRACT

Motivated by the most recent literature, in this paper, we define a new generalization of the Frank matrix that we will call the geometric Frank matrix. In addition to considering some of its algebraic properties, we obtain its LU factorization, its determinant as well as a recurrence relation for its permanent. Upper bounds are set for the spectral norm. We also investigate similar properties for the Hadamard inverse of the geometric Frank matrix. Finally, we provide a MATLAB-R2023a code to facilitate the computation of the permanent of an arbitrary geometric Frank matrix.

***K*eywords** Frank matrix · factorization · determinant · permanent

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APPLYING ENSEMBLE LEARNING TO PREDICT OUTDOOR RADON LEVEL: A CASE STUDY

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ABSTRACT

Radon (the isotope Rn-222) is a radioactive noble gas widely distributed in nature. It is emitted from the Earth's interior and has a short decay half-life of 3.82 days. Due to the properties of radon, fluctuations in its levels are considered an indicator of pre-seismic phenomena, and radon - as a precursor to earthquakes. In this work, the application of ensemble tree learning methods for studying time series with radon emissions data is presented. The Random Forests, and Adaptively resampling and combine (Arcing, Arc-x4) methods were applied. Predictors in the modeling process are rapidly changing meteorological variables such as air temperature, humidity and atmospheric pressure. To construct the models data from two measuring stations located near an earthquake fault in Bulgaria are used. Highly efficient machine learning models for predicting radon concentrations have been built and statistically tested. The influence of predictors in the models has been established. With the help of the obtained models, anomalies in the behavior of radon have been identified, which can be associated with earthquakes that have occurred in the studied geographical area.

***K*eywords** Time series · Radon precursor · Ensemble tree learning · Earthquake · Random Forests

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ICMASE 2025

ON MULTIPLICATIVE JACOBSTHAL NUMBERS AND THEIR COMBINATORIAL PROPERTIES

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ABSTRACT

In this paper, inspired by the researches on multiplicative calculus and their basic multiplicative operations, we define multiplicative Jacobsthal numbers. Furthermore, we give their some interesting combinatorial properties.

***K*eywords** Multiplicative calculus · Jacobsthal numbers

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SUBDIFFERENTIAL DETERMINATION OF A PRIMAL LOWER REGULAR FUNCTION ON A BANACH SPACE

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ABSTRACT

We generalise to Banach space Thibault-Zagrodny Theorem that if f and g are primal lower regular and $\partial f = \partial g$ locally, then f and g locally differ by a constant. Our method consists in reduction to slopes considerations.

***K*eywords** primal lower regular function · slope · determination of a function

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ICMASE 2025

SERVICE LEARNING AS A METHOD FOR TEACHING NUMERICAL ANALYSIS IN CIVIL ENGINEERING

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ABSTRACT

In contemporary civil engineering education, traditional teaching methods often fall short in engaging students or equipping them with the skills necessary to address complex, real-world challenges. This article explores service learning as a pedagogical method for teaching numerical analysis to undergraduate students at the Technical University of Civil Engineering Bucharest (UTCB). Service learning, which integrates academic instruction with community-based engagement, provides a dynamic and meaningful context through which civil engineering students can apply mathematical concepts to socially relevant problems.

The study centers on the application of the least squares method to construct a regression line, a fundamental topic in numerical analysis. Rather than introducing this concept through purely theoretical approaches, students participate in a field-based service project developed in collaboration with local urban planning authorities. The project investigates pedestrian movement near construction zones and its correlation with the distance from public transport stops. Students collect real-world data, apply regression analysis techniques, and interpret the results to assist in the design of safer pedestrian routes and mobility signage.

This case study demonstrates how mathematical modeling becomes more accessible and impactful when embedded within authentic civic contexts. By estimating a linear relationship between pedestrian traffic and transport accessibility, students create a regression model that directly supports urban infrastructure planning. The experience not only strengthens their technical skills in numerical analysis but also fosters essential transversal competences such as teamwork, critical thinking, and civic responsibility.

The outcomes suggest that service learning enhances student engagement, deepens conceptual understanding, and effectively bridges the gap between abstract mathematical instruction and the societal needs of civil engineering practice. The results support the broader implementation of service learning in STEM curricula, particularly in technical universities aiming to integrate academic rigor with sustainable development and community empowerment.

Keywords Service learning · Numerical analysis · Civil engineering education · Regression line · Least squares method · Mathematical modeling · Urban infrastructure

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EULER DETERMINANTS

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ABSTRACT

In 1875, Glaisher systematically found several interesting determinant expressions of numbers, including Bernoulli, Cauchy, and Euler numbers. In this talk, we find several determinants expressing Euler polynomials.

In 2020, Goy and Shattuck presented several determinantal expressions of some families of Toeplitz–Hessenberg matrices with Tribonacci number entries. However, a determinant expression of Tribonacci numbers has not been studied a lot. We give some determinant representations of generalized Tribonacci numbers.

***K*eywords** Euler polynomials · Tribonacci numbers · determinants

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A Comparative Study of Approaches for Solving Lotka-Volterra Equations

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Abstract

In this study, the application of the Hermite collocation method [2-3] to obtain approximate solutions of the Lotka-Volterra equations modelling ecosystem dynamics is investigated. The Lotka-Volterra equations consist of two differential equations describing predator-prey relationships and population dynamics [1], and analytical solutions of these systems are often complex and difficult. Therefore, numerical approximation methods gain importance in the solution of these equations.

In my study, suitable collocation points are selected depending on the dynamic characteristics of the considered system and an approach for the solution of the system with Hermite polynomials is established. The results show that the Hermite collocation method provides sufficient accuracy and computational efficiency for the Lotka-Volterra equations. Moreover, the approximate solutions obtained are presented graphically and analysed in comparison with other numerical methods [4-5].

***K*eywords** Lotka-Volterra equations · Systems of nonlinear differential equations · Hermite collocation method

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ICMASE 2025

$\mathcal{A}$-CHARACTER AND POINT-WISE COUNTABLE FAMILIES

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ABSTRACT

The general concept of a metrizable family of subspaces of a topological space was introduced and applied to the problem of selections of multivalued mapping by M. Choban in [1]. Then in a similar context the concept of a family of subspaces with first axiom of countability was introduced and studied by E. Mihaylova in [3]. Families of p-subspaces and families of $A(k)$-spaces were defined and studied by M. Choban and E. Mihaylova in [2].

All considered spaces are assumed to be Hausdorff.

Fix a non-empty family $\mathcal{A}$ of non-empty subspaces of a topological space X.
A subset H of X is called an $\mathcal{A}$-subset of X if the set H is non-empty, and $H \subseteq L$ for some $L \in \mathcal{A}$.
A subset H of X is called an $\mathcal{A}$-balanced subset of X if the set H is non-empty and $H \subseteq \cap\{L : L \in \mathcal{A}, H \cap L \neq \emptyset\}$.

Definition 1 *The family γ of open subsets of X is called an $\mathcal{A}$-base of the $\mathcal{A}$-subset H in X if $H \subseteq \cap\{W : W \in \gamma\}$ and for every point $x \in H$ and every open subset U of X which contains H there exist an open subset V of X and $W \in \gamma$ such that $x \in V$ and $L \cap W \subseteq U$ for every $L \in \mathcal{A}$ such that $L \cap V \neq \emptyset$. The $\mathcal{A}$-character $\chi_X(H, \mathcal{A})$ of the $\mathcal{A}$-subset H of the space X is the smallest cardinal of the form $|\gamma|$, where γ is an $\mathcal{A}$-base of H.*

Lemma 1 *Let H be a compact $\mathcal{A}$-subset of the space X. Then there exists an $\mathcal{A}$-base γ of H such that:*
1. *$|\gamma| = \chi_X(H, \mathcal{A})$ and $W \cap W' \in \gamma$ for every $W, W' \in \gamma$;*
2. *For every open subset U of X which contains H there exist an open subset V of X and $W \in \gamma$ such that $H \subseteq V$ and $L \cap W \subseteq U$ for every $L \in \mathcal{A}$ such that $L \cap V \neq \emptyset$;*
3. *If $\chi_X(H, \mathcal{A})$ is the countable cardinal $\aleph_0$, then there exists γ such that $\gamma = \{W_n : n \in \mathbb{N}\}$ and $W_{n+1} \subseteq W_n$ for every $n \in \mathbb{N}$.*

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Definition 2 *A family $\mathcal{A}$ of subspaces of a space X is of pointwise-countable type if for every point $x \in \cup\mathcal{A}$ there exists a compact $\mathcal{A}$-balanced subset F of X such that $x \in F$ and $\chi_X(F, \mathcal{A}) = \aleph_0$.*

Theorem 1 *Let $f : X \longrightarrow Y$ be a continuous open mapping of a space X onto a space Y, $\mathcal{A}$ be a family of subspaces of the space Y and $\mathcal{A}^{-1} = \{f^{-1}(L) : L \in \mathcal{A}\}$ be a family of pointwise-countable type of the space X. Then $\mathcal{A}$ is a family of pointwise-countable type of the space Y.*

***K*eywords** Character of a subspace · Point countable type space · Metrizable family of subspaces

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$\mathcal{A}$-CHARACTER AND POINT-WISE COUNTABLE FAMILIES

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ABSTRACT

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Definition 1 *The family γ of open subsets of X is called an $\mathcal{A}$-base of the $\mathcal{A}$-subset H in X if $H \subseteq \cap\{W : W \in \gamma\}$ and for every point $x \in H$ and every open subset U of X which contains H there exist an open subset V of X and $W \in \gamma$ such that $x \in V$ and $L \cap W \subseteq U$ for every $L \in \mathcal{A}$ such that $L \cap V \neq \emptyset$. The $\mathcal{A}$-character $\chi_X(H, \mathcal{A})$ of the $\mathcal{A}$-subset H of the space X is the smallest cardinal of the form $|\gamma|$, where γ is an $\mathcal{A}$-base of H.*

Lemma 1 *Let H be a compact $\mathcal{A}$-subset of the space X. Then there exists an $\mathcal{A}$-base γ of H such that:*
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3. *If $\chi_X(H, \mathcal{A})$ is the countable cardinal $\aleph_0$, then there exists γ such that $\gamma = \{W_n : n \in \mathbb{N}\}$ and $W_{n+1} \subseteq W_n$ for every $n \in \mathbb{N}$.*

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***K*eywords** Character of a subspace · Point countable type space · Metrizable family of subspaces

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ICMASE 2025

FIXED POINT THEOREMS IN PREMETRIC SPACES

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ABSTRACT

We give a characterization of Hausdorff spaces satisfying first axiom of countability using Long orbit or empty value principle. Further, we prove a fixed point theorem in first countable premetric Husdorff spaces without any conditions for compactness or completeness. We obtain a Banach type contraction principle in Σ-semicomplete spaces introduced by Suzuki (2018).

***K*eywords** Multi-valued mappings · Fixed points · Sigma-semicompleteness

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EIGENVECTORS AND EIGENVALUES OF THE EFFECTIVE RESISTANCE MATRIX OF A GRAPH

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ABSTRACT

We consider undirected, weighted and connected graphs on N nodes, whose corresponding graph-related matrices are symmetric. The graph G contains a set $\mathcal{N}$ of N nodes and a set $\mathcal{L}$ of L links. As mentioned in my book [2], I believe that, after the adjacency matrix A and Laplacian matrix Q of a graph G, the effective resistance matrix Ω with elements ω_{ij} is the third important matrix associated with graph G. The effective resistance matrix Ω is closely related to the Laplacian matrix by

$$\Omega = \zeta u^T + u\zeta^T - 2Q^\dagger \tag{1}$$

where u is the all-one vector, the vector $\zeta = \left(Q^\dagger_{11}, Q^\dagger_{22}, \ldots, Q^\dagger_{NN}\right)$ and $Q^\dagger$ is the pseudoinverse of the Laplacian [3], [2, Secion 4.2]. The effective resistance matrix Ω is a distance matrix [2, art. 8]. Here, we explicitly express the eigenvectors $v_1, v_2, \ldots, v_N$ and eigenvalues $\rho_1, \rho_2, \ldots, \rho_N$ of the effective resistance matrix Ω in terms of the eigenvectors $z_1, z_2, \ldots, z_N = \frac{u}{\sqrt{N}}$ and eigenvalues $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_N = 0$ of the possibly weighted, but symmetric Laplacian Q. We also deduce the exact characteristic polynomial and thus improve on a famous interlacing result by Fiedler [1, Corollary 6.2.9], [2, Theorem 33].

Keywords Graphs · Spectrum · Electric currents

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ICMASE 2025

DIOPHANTINE TRIPLES IN LINEAR RECURRENCES

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ABSTRACT

A Diophantine $m-$ tuple is a set of m distinct positive integers $\{a_1, a_2, \ldots, a_n\}$ such that $a_i a_j + 1$ is a square for all $1 \leq i < j \leq n$. If $m = 3$, it is called Diophantine triples. For example $\{1, 3, 8\}$ is a Diophantine triples which are consecutive Fibonacci numbers. The Fibonacci numbers satisfy the recurrence relation

$$F_n = F_{n-1} + F_{n-2}$$

for ≥ 2 with initials $F_0 = 0$ and $F_1 = 1$. More generally, the set $\{F_{2n}, F_{2n+2}, F_{2n+4}\}$ is a Diophantine triples. The companion sequence of Fibonacci is known Lucas sequence $\{L_n\}$ safisfies the same recurrence with initial conditions are $L_0 = 2$ and $L_1 = 1$. The equation system

$$\begin{aligned} ab + 1 &= E_x \\ ac + 1 &= E_y \\ bc + 1 &= E_z \end{aligned}$$

was solved by Luca ans Szalay where E_n is n^{th} Fibonacci and Lucas number. In this talk, we give the details about other linear recurrences and Diophantine triples.

***K*eywords** Diophantine triples, linear recurrence, Fibonacci numbers with generalizations.

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APPROXIMATION BY NEW (λ, μ)-BERNSTEIN-KANTOROVICH OPERATORS

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ABSTRACT

Using the shape parameter λ is of great interest nowadays. This parameter provides considerable flexibility, and improves the modeling possibilities in approximation theory. Two main studies can be given as [2] and [3]. Inspired by these studies, using the Kantorovich operators in [1] and the idea of [3], we generalize another variant of (λ, μ)-Bernstein-Kantorovich operators. This talk presents new generalized (λ, μ)-Bernstein-Kantorovich operators. We investigate the approximation properties of these new generalized operators. We also provide some numerical and graphical examples to demonstrate the rate of convergence of the constructed operators.

***K*eywords** λ-Bernstein operators · (λ, μ)-Bernstein-Kantorovich operators · Rate of convergence

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DISCOVERING POINT GROUPS BY USING MODULAR ORIGAMI

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ABSTRACT

It is well-known that there exist numerous connections between origami and mathematics, from basic geometrical shapes, over geometric constructions using paper-folding, combinatorics and graph theory, up to abstract algebra and number theory [4, 6, 7]. The idea to use origami in teaching and popularizing mathematics and chemistry is natural, in particular at the level of primary education. Still, research in this area is scarce and so far has been limited to teaching basic geometry [2, 5, 10], effects on mathematical thinking in general [11], and teaching visualisation of chemical structures [3]. Some of these studies also include some symmetry considerations, but mostly limited to mirror (reflection) symmetry.

On the other hand, the theory of symmetry groups is fundamental for crystallography (and stereochemistry). Symmetry group theory has in fact historically developed from crystallographic considerations of symmetries of crystals and crystal structures [1]. Macroscopic crystals (as well as VSEPR models of molecule geometry) are modelled by various polyhedra with huge importance of their point symmetry groups, and crystal structures can be modelled by periodic tesselations and classified according to symmetry into space groups. The usual way to teach the necessary symmetry theory is using cardboard models, so the idea that origami could be used to teach symmetry is quite obvious.

Modular origami is an origami technique in which the final model is assembled from parts (modules) which are usually much easier to fold than the usual origami models. It allows a variety of examples of polyhedra with different types of point symmetry to be folded quickly, cheaply and easily [9], and has several advantages with respect to the usual cardboard and other solid models. In this presentation, we aim to introduce a method to teach fundamental point group theory concepts (recognition of various types of symmetry, the notion of point groups and their subgroups) using modular origami. This shall be done in an interdisciplinary fashion, combining mathematical abstract thinking and geometrical reasoning with its applications in crystallography and stereochemistry. More specifically, the ideas of the approach shall be demonstrated by making the well-known modular origami model known as Jackson cube [8], first in one color to investigate the full point group of the cube (O_{h}), and then by exchanging modules make it multicoloured to discover its subgroups.

***K*eywords** mathematics education · symmetry · origami · mathematical crystallography · mathematical chemistry · hands-on activities in teaching mathematics · group theory · point groups

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PREDICTIVE MODELING OF IT CANDIDATE SELECTION USING LOGISTIC REGRESSION: A COMPETENCY-BASED APPROACH

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ABSTRACT

The increasing complexity of technical roles in the modern IT industry has amplified the need for structured, data-driven approaches to recruitment. As companies strive to align talent acquisition processes with performance-driven outcomes, the integration of statistical modeling into candidate evaluation has emerged as a compelling solution. This study investigates the application of binary logistic regression as a predictive tool for assessing the likelihood of candidate selection for IT Developer positions, based on a dataset comprising standardized performance scores across both technical and cognitive competencies.

The research utilizes a real-world dataset containing individual scores in PLSQL, SQL, Critical Thinking (CT), and Problem Solving (PS), collected as part of a candidate evaluation process. These scores were normalized to ensure consistency and comparability across individuals. The logistic regression model was constructed using the maximum likelihood estimation method to determine the coefficients corresponding to each predictor variable. The dependent variable in the model is binary, representing whether a candidate was selected ("1") or not selected ("0").

Each coefficient in the model reflects the marginal effect of a specific skill on the log-odds of being selected, enabling a nuanced understanding of how different competencies contribute to the final hiring decision. By computing the logistic function for each candidate, the model yields a probability score between 0 and 1, which represents the predicted likelihood of selection. These probabilities are then used to rank candidates and assess the alignment between the model's predictions and the actual selection outcomes.

The analysis reveals several important insights. Technical skills (PLSQL and SQL) exhibit a strong positive influence on the probability of selection, while cognitive abilities, particularly Critical Thinking, also play a significant role. Interestingly, the Problem Solving variable displayed low variance in the original dataset, limiting its contribution to the model but highlighting a potential area for improvement in candidate assessment procedures. Visualizations such as histograms, scatter plots, and correlation matrices were employed to enhance the interpretability of the results.

The paper further evaluates the model's predictive accuracy by comparing the predicted selection statuses to actual outcomes and discussing the potential sources of discrepancies. This evaluation provides a foundation for refining selection criteria and adjusting model parameters for future iterations.

From a practical perspective, the study underscores the value of implementing logistic regression models in human resource management, particularly in technical fields where candidate differentiation is both subtle and critical. By shifting towards evidence-based decision-making frameworks, organizations can increase fairness, transparency, and efficiency in the recruitment process.

This research contributes to the interdisciplinary field of data science and human resources by demonstrating the viability and benefits of predictive modeling in talent acquisition. It also opens

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avenues for future work, such as the integration of additional variables (e.g., soft skills, prior experience), the use of alternative classification algorithms (e.g., decision trees, support vector machines), and the development of automated recruitment decision-support systems.

***K*eywords** Logistic regression · Candidate selection · Predictive modeling · IT recruitment · Human resource analytics · Technical and cognitive assessment

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A Research-Based Approach to Teaching Mathematics for Engineers Using the Case Method

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Abstract

In modern engineering education, mathematical knowledge must serve not only as a theoretical foundation but also as a practical tool for solving complex real-world problems. However, traditional teaching methods in mathematics often rely on passive transmission of information and emphasize abstract computation over contextual understanding. This paper explores a research-based pedagogical alternative, the Case Method, as a transformative approach to teaching mathematics to engineering students. Rooted in constructivist learning theory and active learning principles, the Case Method engages students in realistic scenarios that demand analytical thinking, mathematical modeling, and collaborative problem-solving.

By integrating mathematics instruction with authentic engineering cases, the Case Method facilitates deeper cognitive processing, enhances motivation, and fosters long-term knowledge retention. This approach shifts the role of the instructor from lecturer to facilitator and challenges students to take ownership of their learning. Through guided exploration, they apply core mathematical concepts, such as linear systems, differential equations, optimization, and numerical methods, to address challenges drawn from disciplines such as civil, mechanical, and electrical engineering.

The paper provides a theoretical framework for using the Case Method in STEM education, supported by findings from the literature on research-based teaching. It presents concrete examples of classroom implementation, highlighting how mathematical concepts can be taught through structured case studies involving engineering scenarios such as heat transfer, structural stability, or construction logistics optimization. The observed impact on student engagement, critical thinking, and communication skills confirms the method's value in developing both academic competencies and professional attitudes.

While the Case Method presents certain challenges, such as increased preparation time and the need for flexible assessment methods, it aligns closely with the goals of modern engineering education, which prioritize problem-solving, interdisciplinary thinking, and applied knowledge. The study concludes that embedding the Case Method within mathematics courses contributes significantly to preparing future engineers for the demands of their profession, offering a powerful synthesis between theoretical rigor and practical relevance.

***K*eywords** Case Method · Engineering education · Mathematics teaching · Research-based learning · Applied mathematics · Active learning · Problem-based instruction

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ICMASE 2025

THE DISTRIBUTION OF $\alpha p^{\gamma} + \beta$ MODULO ONE AND r-FREE NUMBERS

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ABSTRACT

Let $\alpha, \beta \in \mathbb{R}$ be given. Assume that $a_1, a_2, \ldots, a_n$ are distinct positive integers and let $\mathcal{P}$ be the infinite set of prime numbers for which $a_1, a_2, \ldots, a_n$ does not form a reduced residue system modulo p^r, $r > 2$ for any $p \in \mathcal{P}$. For any fixed γ with $0 < \gamma < 1$, we prove that there are infinitely many primes $p \in \mathcal{P}$ such that $||\alpha p^{\gamma} + \beta|| < p^{-\theta}$ and all the numbers $p + a_1, \ldots, p + a_s$ are free of rth powers.

Keywords Distribution modulo one · r-free numbers · esti-mates of exponential sums

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ICMASE 2025

ON CHARACTERIZATION OF A TYPE OF SEMI-SYMMETRIC METRIC CONNECTION ON A SPACETIME

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ABSTRACT

General relativity, which has disclosed the basic connection between physics and the geometry of spacetimes, is one of the most successful physics theories of twentieth Century. In addition to its crucial importance in theoretical studies, general relativity has found success in technology when applied to our daily lives.

A time-oriented, connected and four-dimensional Lorentzian manifold was modeled by both general relativity spacetime and cosmology.

The geometry of Lorentzian manifolds is used to investigate the behavior of vectors on the manifold. Lorentzian manifolds are emerging as the most effective study model to explain the general relativity.

A spacetime is a Lorentzian manifold M with the Lorentzian metric g of signature $(-,+,+,+)$ which permits a globally timelike vector field. Different types of spacetimes have been studied in various ways, such as ([1]-[5]).

Let (M,g) be a connected and time-oriented Lorentzian manifold. If $M=-I\times_{\phi^2}M^*$, where I is the open interval of $\Re$, ϕ is a smooth function (or scale factor, or warping function) and M^* is considered as a three-dimensional Riemannian manifold, then M is named as a generalized Robertson-Walker spacetime [6]. If we consider that M^* is a three-dimensional Riemannian manifold with constant curvature, then the generalized Robertson-Walker spacetime reduces to Roberston-Walker spacetime. Some examples of special spacetimes related by generalized Robertson-Walker spacetimes are de Sitter spacetime, Einstein de Sitter spacetime, static spacetime and Friedmann cosmological models.

A perfect fluid spacetime is a four-dimensional spacetime whose non-vanishing Ricci tensor R_{ij} is of the form

$$R_{ij}=\alpha g_{ij}+\beta w_i w_j$$

assuming that α, β are smooth functions, g is the Lorentzian metric and w_i is the velocity vector satisfying the condition $w_i w^i=-1$.

In a perfect fluid spacetime, the energy momentum tensor T_{ij} is given by

$$T_{ij}=(p+\sigma)w_i w_j+pg_{ij}$$

where σ and p denote the energy density and the isotropic pressure, respectively and w_i is a non-vanishing vector.

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The Einstein field equations without cosmological constant are presented by

$$R_{ij} - \frac{R}{2} g_{ij} = kT_{ij}$$

where R denotes the scalar curvature and k indicates the gravitational constant.

Let $\bar{\nabla}$ represent a linear connection on a Lorentzian manifold (M, g) of dimension n. This connection is referred to a semi-symmetric if the torsion tensor

$$\bar{T}^k_{ij} = \delta^k_j w_i - \delta^k_i w_j$$

where w_i is the associated vector of the linear connection $\bar{\nabla}$.

A linear connection $\bar{\nabla}$ is said to be metric if $\bar{\nabla}_k g_{ij} = 0$, otherwise it is non-metric. Hayden [7] studied the properties of the metric connection on a Riemannian manifold. A systematic study of semi-symmetric connection on a Riemannian manifold was given by Yano [8]. Recently, De et al. [9] have studied the properties of Lorentzian manifolds endowed with the semi-symmetric metric connection.

The main purpose of this paper is to investigate the general properties of spacetime with a semi-symmetric metric connection. Firstly, we show that a spacetime equipped with a semi-symmetric metric w-connection is a dark matter if the Ricci tensor with respect to the connection $\bar{\nabla}$ vanishes. Also, we prove that if the curvature tensor of such a spacetime with respect to the linear connection vanishes then this spacetime is conformally flat.

In the other parts of this paper, we prove that such a spacetime under some conditions becomes Robertson-Walker spacetime or Yang-Pure spacetime or de Sitter spacetime. Finally, we apply these spacetimes to general relativity and we discuss their physical interpretations of some geometric results.

***K*eywords** Generalized Robertson-Walker spacetime · perfect fluid · general relativity · semi-symmetric metric connection · quasi-constant curvature · conformal curvature tensor.

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CONTINUITY OF SOLUTIONS OF PARAMETRIC OPTIMIZATION PROBLEMS

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ABSTRACT

Parametric optimization problems are tasks in which the objective function and/or the domain over which optimization occurs change depending on a parameter. The main subjects of study in these problems are the optimal value function and the multivalued mapping of the solutions to the optimization problem, which depend on the parameter.

Examining such problems is significant in the general theory of optimization and operations research due to their direct connection with the sensitivity and stability of optimization tasks under various disturbances. In practical problems, inaccuracies inevitably arise in observing a given process, as well as errors in the numerical processing of the obtained data. Therefore, it is necessary to know whether the obtained numerical solution to the disturbed optimization problem is close (in some sense) to the solution of the original problem.

Such tasks naturally arise in game theory and mathematical economics. In a standard setup, the optimal consumption of a given agent is sought depending on their income and market prices. Another typical situation is when an optimal investment strategy is considered as a function of market indices.

A key tool in parametric optimization problems is Berge's maximum theorem. This theorem provides a sufficient condition for the continuity of the optimal value function and the upper semicontinuity of the solutions to the optimization problem. One important application of this theorem is in obtaining equilibrium existence results in game theory.

A substantial part of the study of such problems involves working with multivalued mappings. The sets over which we optimize are given as values of a multivalued mapping with the parameter as its argument. The set of solutions is also a multivalued mapping that has the parameter as its argument. We are mainly interested in the continuity properties of these multivalued mappings, specifically in what assumptions to impose on the mapping and what conclusions we would like to obtain for the set of solutions.

The continuity of multivalued mappings is a significantly more complex and ambiguous concept than the continuity of functions. The definition and study of the notion of continuity initially belonged to theoretical mathematics. Continuously developing fields of applied mathematics, such as optimization, economics, and others, further stimulate the expansion of these constructions, addressing the need for such a mathematical apparatus through which broader concepts can be established and more global results can be derived. Analogous to the relationship between limits of sequences and limits of functions, theories of convergence of sequences of sets and continuity of multivalued mappings have developed in parallel. The concepts of internal and external limits of a sequence of sets were first introduced by the French mathematician Paul Painlevé (1863-1933) in 1902, with the convergence of a sequence of sets characterized by the coincidence of its internal and external limits. Later, after the first quarter of the 20th century, scholars like Felix Hausdorff (1868-1942)

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and Kazimierz Kuratowski (1896-1980) developed and popularized Painlevé's concept, basing their work on it. This is why today this convergence is known as Painlevé-Kuratowski convergence.

With sets as values, a multivalued mapping allows for a similar construction to be applied to it. Thus, the concept of Painlevé-Kuratowski continuity of a multivalued mapping is reached. Another construction, which combines well with the metric in a metric space, was introduced and studied by mathematicians Pompeiu (1873-1954) and Hausdorff.

Another type of continuity we will consider in our work aligns well with the topological structure of the space and can be defined for mappings between arbitrary topological spaces. In our work, we will call this type of continuity topological.

It is established that in a metric space with compact images, the three types of continuity of a multivalued mapping coincide. It is precisely compact images that are required by Berge's theorem. The relaxation of the compactness condition while preserving the conclusion in Berge's theorem is a question explored by various mathematicians. In a recent article by Feinberg et al., this problem is considered in general topological spaces.

In our study, we deal with this issue when the space over which we optimize is metric. Our main focus is proving the upper semicontinuity of the multivalued mapping of the solutions to an optimization problem. We also examine the interrelationships between different conditions imposed to relax compactness. We obtain several results that we have not encountered in the literature, summarizing and placing in a single framework known theorems in this field. In the beginning we optimize the objective function over a finite-dimensional space, with a weak type of continuity (Painlevé-Kuratowski) of the multivalued mapping additionally imposing a condition to restrict the behavior of the function at infinity. In a case where the objective function does not depend on the parameter, we introduce a new concept of well-posedness. We use it to obtain a general result for which we additionally require only the continuity of the multivalued mapping in the sense of Painlevé-Kuratowski and the continuity of the objective function. After this we provide sufficient conditions for the well-posedness of a pair. We show how known results are obtained as special cases. During our considerations, we introduce a new type of lower semicontinuity, stronger than all known to us so far.

When the objective function depends on the parameter, we obtain a theorem, which provides both new (to our knowledge) results and generalizes Berge's Theorem in its part concerning the semicontinuity of the solutions, as well as other previous results.

After this we obtain so-called inverse results. They address the question: given the upper semicontinuity of the solutions for all objective functions of a certain type, what can we say about the other objects in the problem, for example, the multivalued mapping. Most authors focus on proving the upper semicontinuity of the solutions. This is because, when we have upper semicontinuity of the mapping at some point, we know that for other points that are in some sense close to all solutions to the optimization problem at these points are close to solutions at but not to all solutions at. When we have lower semicontinuity of the mapping we know that at points close to the solutions to the optimization problem approximate well all solutions obtained at but there may also be those that are very far from the solutions at. Still, the preference is to have solutions close to those of the original problem though possibly not encompassing all its solutions, rather than having a good approximation of all solutions to the original problem but with the potential problem of some of the obtained solutions being very far from the solutions to the original problem.

***K*eywords** Optimization problems · Parametric problems · Objective function · Berge's theorem

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ICMASE 2025

ON FREE ASSOCIATIVE ALGEBRAS WITH ADITIONAL ACTION

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ABSTRACT

It is a well known result of Emmy Noether [4, 5] that the algebra of invariants of the polynomial algebra in d commuting variables under the action of finite group G is finitely generated. In the noncommutative case, this is only true for finite groups, consisting of scalar matrices. However, equipped with an additional action, defined by Koryukin in [R5], the algebra of invariants of polynomials in d non commuting variables becomes finitely generated. In [1, 2] we study the invariants of free associative algebras under the action of fixed finite groups.

***K*eywords** Free associative algebras · Noncommutative symmetric polynomials · Finite generation · Koryukin action

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ON SOME CLASSES OF GENERALIZED QUASI-EINSTEIN SPACETIMES WITH APPLICATIONS IN GENERAL RELATIVITY

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ABSTRACT

In general relativity, a spacetime, denoted by M^4 is viewed as a Lorentzian manifold equipped with a Lorentzian metric g. This metric has a signature of $(-,+,+,+)$, indicating the mixture of positive and negative signs. It allows for the presence of a vector that is time-oriented and valid globally throughout the spacetime.

A Lorentzian manifold M^4 is called a perfect fluid spacetime [1], if the non-zero Ricci tensor R_{ij} satisfies

$$R_{ij} = \alpha g_{ij} + \beta A_i A_j$$

where α, β are scalar functions and A_i is the non-zero 1-form named as "the generator of the manifold" and $A_i A^i = -1$.

For a perfect fluid spacetime, the energy momentum tensor T_{ij} is given by [2]

$$T_{ij} = (p+\sigma)A_i A_j + p g_{ij}$$

where σ and p denote the energy density and the isotropic pressure, respectively and A_i is a non-vanishing vector.

The Einstein field equations without cosmological constant are presented by

$$R_{ij} - \frac{R}{2} g_{ij} = kT_{ij}$$

where R denotes the scalar curvature and k indicates the gravitational constant.

In modern cosmology, dark energy is considered as a candidate to accelerate the expansion of the universe and the scalar functions σ and p are considered by an equation of state (EoS), $p = p(\sigma, T_0)$ that regulates the quality of the ideal fluid by denoting T_0 as the absolute temperature. If we take T_0 as a constant, then the (EoS) is reduced to $p = p(\sigma)$. Then, this spacetime is called isentropic [3].

From (EoS), a perfect fluid spacetime is referred as stiff matter if $p = \sigma$, dark matter era if $p = -\sigma$, dust matter era if $p = 0$, the radiation era if $p = \frac{\sigma}{3}$, cosmic walls if $p = -\frac{2\sigma}{3}$ and strings if $p = -3\sigma$ ([4]-[6]). If $\frac{p}{\sigma} < -\frac{1}{3}$ then the universe represents accelerating phase, if $-1 < \frac{p}{\sigma} < 0$ then the universe represents quintessence phase.

A generalization of Einstein manifolds is the generalized quasi-Einstein manifold ([7],[8]). The Ricci tensor of a generalized quasi-Einstein manifold satisfies the following condition

$$R_{ij} = \alpha g_{ij} + \beta A_i A_j + \gamma(A_i B_j + A_j B_i)$$

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where γ is a scalar , A_i is unit a time-like vector and B_i is a unit space-like vector. Also, A_i and B_i are orthogonal vectors. A Lorentzian manifold whose Ricci tensor satisfies the last equation is called generalized quasi-Einstein spacetime.

In this paper, some special conditions in a generalized quasi-Einstein spacetime are considered. Under some special conditions, the physical properties of these spacetimes are examined. Then, it is shown that such a turns into a perfect fluid or static spacetime or a special type of product spacetime under some assumptions.

In the last part of this study, the applications of the considered spacetime in general relativity are discussed.

***K*eywords** Generalized quasi-Einstein spacetime · perfect fluid · static spacetime · general relativity · expansion scalar · vorticity.

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ICMASE 2025

ASYMPTOTIC REGULAR CONTRACTIONS ON MULTIPLICATIVE METRIC SPACES

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ABSTRACT

In this paper, we study on asymptotic regular contraction mapping and multiplicative metric spaces. Asymptotic regular contraction mapping and asymptotic regular cyclic contraction mapping are defined on multiplicative metric spaces. Then, using these mappings new fixed point theorems were obtained and proved in multiplicative metric spaces. Finally, an application is presented to make the subject clearer and more understandable.

***K*eywords** Multiplicative metric · Asymptotic regular contraction mapping · Fixed point

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FIXED POINT THEOREMS ON DIGITAL RECTANGULAR METRIC SPACE

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ABSTRACT

In this study we focus on digital images with κ adjacency relation. We define rectangular metric on digital images. By using the differences related with convergency, Cauchy sequence and κ continunity in digital images and by using rectangular property we obtain and prove Caristi and Kannan type fixed point theorems.

***K*eywords** Digital Image · Rectangular metric · Fixed point

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INVESTIGATION OF THE STABILITY PERFORMANCE OF AN UNMANNED AERIAL VEHICLE (UAV) WITH BOTH DOUBLE DIHEDRAL AND DOUBLE SWEPT MORPHING WING

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ABSTRACT

Fixed-wing Unmanned Aerial Vehicles (UAVs) are an aircraft concept that has been the focus of recent studies in terms of stability and control. The stability of fixed-wing UAVs with morphing aerodynamic components has also been extensively studied. The stability and control performance of the UAV changes as a result of the morphing of the control surfaces or wing geometry. In extreme conditions, the cruise flight of the UAV is out of equilibrium and the UAV is subject to disturbances. As a result, the UAV is planned to provide a stable action in accordance with the design requirements and strives to return to the equilibrium position from which it moved away as a result of the disturbance. In other words, the aircraft has a stable behavior. If the aircraft returns to the equilibrium position without using control surfaces, this is called fixed stick motion and the aircraft is designed to return to the equilibrium position without using any trim surfaces. In this study, the effect of changing the wing structure on the stability performance of a UAV is investigated. Changes in the damping ratio value and undamped natural frequency values of the UAV with double dihedral and double swept angles on the wing will be analyzed. This work also has been supported by Erciyes University Scientific Research Projects Coordination Unit under grant number FDK-2025-14354.

Keywords · Double Dihedral · Double Swept · Morphing Wing · UAVs

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FLOW CONTROL AND PERFORMANCE ENHANCEMENT OF VTOL UAVS USING VORTEX GENERATORS AND BLOWING TECHNIQUES: A NUMERICAL APPROACH

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ABSTRACT

In this study, the aerodynamic performance of a vertical take-off and landing (VTOL) unmanned aerial vehicle (UAV) is investigated through the application of both active and passive flow control techniques. Passive control is achieved via vortex generators, while active control is implemented using blowing methods. A novel UAV design incorporating both strategies has been developed, and multiple configurations have been evaluated using Computational Fluid Dynamics (CFD) simulations. The main objective of this work is to enhance the overall flight performance of the UAV by delaying flow separation, controlling the boundary layer, and increasing aerodynamic efficiency. Various geometrical and operational parameters such as blowing magnitude, speed, angle, location and shape and size of the blowing area, as well as the shape, size and placement of the vortex generators were explored. In the first stage, each of these parameters was individually implemented and tested on the VTOL UAV design to assess its influence on aerodynamic performance. In addition, various combinations of these parameters were systematically investigated to evaluate their interactive effects and overall contribution to flow control efficiency. Preliminary results demonstrate that the integrated use of active and passive methods significantly improves aerodynamic performance, especially in transition regimes and low-speed operations. The proposed research provides important scientific insight contributions to the aerodynamic optimization of modern VTOL UAVs. This work also has been supported by Erciyes University Scientific Research Projects Coordination Unit under grant number FDK-2025-14354.

***K*eywords** Active flow control method · passive flow control method · UAV · VTOL · Vortex Generator · Blowing Method · CFD · Performance Optimization · Separation Control

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ON SOFT MULTIPLICATIVE NORMED SPACES

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ABSTRACT

In this paper, soft normed spaces is considered with different perspective soft multiplicative norm is defined by using soft points and then convergent sequences, Cauchy sequences are stated in soft multiplicative normed spaces. Some new fixed point theorems are obtained and proved in soft multiplicative normed spaces.

***K*eywords** Soft norm · Multiplicative norm · Soft Fixed point

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HIGH ACCURACY NUMERICAL SOLUTION OF AN ADVECTION–ADSORPTION MODEL WITH FREUNDLICH OR LANGMUIR ISOTHERMS

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ABSTRACT

For the numerical solution of a system of nonlinear advection-adsorption partial differential equations, we develop a semi-implicit finite difference scheme with a second-order spatial discretization error. The mathematical problem arises from the domain of water purification using activated carbon. We are particularly focused on a use-case involving the simulation of fixed-bed adsorption process for the removal of organic micropollutants from drinking water. The implicit approximation of part of the system's equations is needed due to some specifics of the values of model parameters. Our aim is to accurately simulate the outflow concentrations of the adsorbates. This, in turn, allows for the precise computation of breakthrough curves, which are key indicators in evaluating the effectiveness of water treatment.

The mathematical model is capable of describing both single-component and multi-component adsorption processes. In the single-component adsorption scenario, we examine two formulations of the model—one utilizing the Freundlich isotherm and the other based on the Langmuir isotherm to represent the adsorption term. Incorporating various isotherm equations into the mathematical model enhances its flexibility and enables the simulation of a broader range of adsorbate–adsorbent systems. In the multi-component case, the numerical method is effectively applied to simulate breakthrough curves for up to ten contaminants.

The proposed finite difference scheme is analyzed through numerical experiments, demonstrating its effectiveness across various parameter settings and confirming second-order convergence with respect to spatial discretization. A comparison is made with a first-order numerical scheme applied to the same problem, highlighting the advantages of the newly proposed method. In particular, using the latter, it is shown that the maximum approximation error for the concentration of contaminant, relative to the inlet concentration, does not exceed 10^{-6} in magnitude.

Keywords advection-adsorption · numerical simulation · breakthorugh curve · Freundlich isotherm · Langmuir isotherm

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THE LANA MODEL: AGENT-BASED MODEL USED FOR SIMULATION OF SIGNAL TRANSMISSION IN NEURONS

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ABSTRACT

Since the Hodgkin-Huxley model was published, the differential equation has been applied to describe the dynamics of signal transmission in the brain. Various scientists are working on simplifying it, taking into consideration the demonstration's assurance of accuracy for the biophysical process in the brain.

Regarding the interaction of millions of neurons in the brain, one of the convenient ways to modulate and simulate their interaction is through agent-based modeling due to its possibility to adjust received parameters. The advantage of the agent-based model is that it allows adjusting one parameter while keeping the other parameters fixed, enabling the investigation of individual effects and testing scientific hypotheses related to signal transmission in the brain. In this study, the agent-based model, the Lana model, is presented, which is based on the generalization of undergoing bio-physical processes during signal transmission in the brain. In the Lana model, processes are grouped into two subsets (agents) describing either the signal (agent Signal) or the neuron-related processes necessary for the signal transmission (agent Neuron). Moreover, the neuron is observed with all bio-physical properties using the mathematical equation, the modified Hodgkin-Huxley model. The Agent neuron in the Lana model has the following properties: functionality, trigger threshold, number of functional accesses (dendrites), output range (axons), number of functional outputs (telodendrions), predisposition of the receiving signal, and recovery time. In addition, the Agent signal in the mentioned model has the following properties: signal identification, current strength, repeated signal frequency, initial position, and radius of the affected neuron. Using these two agents, the Lana model enables the simulation of neuron signal transmission in the brain, as well as the flexibility that enables it. The signal transmission is modulated using both agents while the agent Neuron simulates changes in flexibility. The Lana model is considered successful if it can produce the spike graph or pathway from the MRA image.

The purpose of the Lana model lies in the application and regulation of neurons in both physiological and pathological states. The mentioned type of application and regulation of the signal in neurons can play a significant role in understanding neurological diseases, such as stroke, Multiple sclerosis, and Parkinson's disease, as well as in future treatments for pathological brain states.

Keywords Signal Transmission · Agent-based model · Neurons

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SOFT SET-BASED REPRESENTATION OF TIME SERIES AND ANALYSIS

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ABSTRACT

Soft set theory, originally introduced by Molodtsov in 1999 in [4], presents a powerful and flexible mathematical framework for dealing with uncertainties, imprecisions, and vagueness commonly encountered in real-world problems. Its parameterized structure allows it to effectively model situations in which traditional mathematical tools, such as fuzzy sets or probability theory, may face limitations. Despite its widespread applications across fields such as decision-making, engineering, medical diagnosis, and economics, its integration with time series analysis remains relatively underexplored.

The aim of this study is to bridge this gap by proposing two different methods to represent time series data within the soft set framework: the Frequency Threshold Soft Set and the Recurrence Threshold Soft Set. These approaches offer different perspectives for identifying cyclic behaviors or steady patterns and for highlighting dominant periodicities or oscillations in time-dependent data.

To validate the proposed methods and the associated similarity measure, a real or simulated time series dataset is used as a case study. The dataset is processed using both soft set representations, and similarities between different time intervals or recurring patterns are computed using the soft set-based similarity measure. The results are then analyzed and discussed in terms of their interpretability and their effectiveness in capturing meaningful temporal structures.

Keywords Soft Sets · Soft Set-Based Representation of Time Series · Application of Soft Sets

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ON COMPACT EXPLICIT FORMULAS OF THE PARTIAL FRACTION DECOMPOSITION. APPLICATIONS

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ABSTRACT

Partial fraction decomposition is a fundamental topic with applications in various fields of mathematic such that calculus, differential equations, control theory, and other fields of applied mathematics and other subject areas. A well known theoretical theorem in algebra asserts that every rational function has unique partial fraction decomposition. Several methods have been improved in the literature for computing the partial fraction decomposition. Despite that, this topic continues to attract much attention, and there have been recent developments in the computation aspect for general rational functions, as well as for some special. Meanwhile, the approaches and methods for decomposing a rational function into partial fractions are computationally intensive, especially when the multiplicities of roots of the denominator are higher.

This talk concerns some new explicit compact formulas of the partial fraction decomposition. We provide an approach for the partial fraction decomposition of the functions $f(x) = R(x)/Q(x)$, where $R(x)$, $Q(x)$ are polynomials in $\mathbb{R}[X]$ or $\mathbb{C}[X]$, which are mutually prime, such that the degree of r is less than the degree of Q. The essence of our approach requires a computational process based on some known results. New results are provided and other are recovered. Finally, some applications and illustrative examples are given, in order to show our new approaches.

***K*eywords** Partial fraction decomposition, Compact formulas, Multiple poles,

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INNOVATIVE GRAPHS VIA E_J-NEIGHBORHOODS RISING FROM IDEALS

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ABSTRACT

Neighborhoods systems is an important tool for graphs in view of topological. So, in this paper, we establish new types of eight neighborhoods via the notion of set-ideal and E_j-neighborhoods from vertices of any graph. Then, the accuracy measures and boundary regions of these approximations are calculated. We explore novel types of j-lower (resp. j-upper) approximations are obtained and some algorithms are introduced. Finally, we give a real-life problem which is related to our methods.

***K*eywords** Neighborhood Systems · Rough Sets · Graph Topology · j-Accuracy Measure

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ABSORBABLE GROUPS

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ABSTRACT

In this study, we provide an innovative concept in group theory that we call it an absorbable group. We discuss some of its characteristics as well as how it relates to well-known ideas such as solvable groups and nilpotent groups. Due to their well-behaved structure and hierarchical breakdown, nilpotent groups are an important class of groups in group theory. If the index [G: H] is a prime number, then G is said to have a prime index. Prime index subgroups are crucial in the context of nilpotent groups. For instance, H must be normal in G if G is a nilpotent group and H is a subgroup with prime index p. Additionally, group solvability and automorphism group structure are impacted by the interaction between nilpotent groups and their prime index subgroups

Keywords Absorbable group · Solvable groups · Nilpotent groups

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ON FIBONACCI POLYNOMIALS AND DETERMINANTS

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ABSTRACT

The classical sequence of Fibonacci polynomials is a special case of Chebyshev polynomials and has been studied in various fields. This class of polynomials, denoted by $\{F_n(x)\}_{n\geq 0}$, is defined in [2] by the initial conditions $F_0(x) = 0$, $F_1(x) = 1$, and the recurrence relation: $F_{n+1}(x) = xF_n(x) + F_{n-1}(x)$, for $n \geq 1$, as well as by the following identity:

$$F_{n+1}(x) = E_n(x,-1) = \sum_{m=1}^{\infty} \binom{n-m}{m} x^{n-2m} = \det \begin{pmatrix} x & -1 & 0 & 0 & \cdots \\ 1 & x & -1 & 0 & \cdots \\ 0 & 1 & x & -1 & \cdots \\ \vdots & \vdots & \ddots & \ddots & -1 \\ 0 & 0 & \cdots & 1 & x \end{pmatrix}_{n\times n}. \quad (1)$$

The Identity (1) was derived from the second-type Dickson polynomials presented in [3]. In [2], several identities for Fibonacci polynomials were obtained using matrix theory. Moreover, various properties and generalizations of this well-known sequence can be found in the literature (see, for example, [1], [5], [4], and [6]), along with numerous applications in algebra, analysis, combinatorics, and matrix theory. For instance, in [5], a new procedure for the numerical solution of boundary value problems involving expansions in Fibonacci polynomials was introduced. The fundamental Fibonacci system, as introduced in [6], is defined by the recurrence relation

$$F_n^{(s)} = F_{n-1}^{(s)} + F_{n-2}^{(s)} + \cdots + F_{n-r}^{(s)}, \quad n \geq r, \quad (2)$$

where the initial conditions are given by $F_n^{(s)} = \delta_{s-1,n}$, for $0 \leq n \leq r-1$. The properties of the fundamental Fibonacci system and the fundamental solution were established, and new identities for generalized Fibonacci numbers were derived. Considering these previous results, this work focuses on a generalization given by the following linear difference equation of order $r \geq 2$:

$$F_n^{(s)}(x) = xF_{n-1}^{(s)}(x) + \sum_{i=1}^{r-1} F_{n-i-1}^{(s)}(x), \quad \forall n \geq r, \quad (3)$$

with initial conditions given by $F_{s-1}^{(s)}(x) = 1$, $F_n^{(s)}(x) = 0$, for all $x \in \mathbb{R}$, $0 \leq n \neq s-1 \leq r-1$. We refer to the class of polynomials defined by Equation (3) as the Fibonacci polynomials.

Our goal is to study Equation (3) through the fundamental Fibonacci system and Chebyshev polynomials, that is, a determinantal approach to the terms of this polynomial sequence. This method allows us to establish new properties and identities for this newly generalized class of polynomials.

Fibonacci polynomials, a special case of Chebyshev polynomials, can be expressed as determinants of structured matrices, such as Jacobi or tridiagonal matrices. These representations make explicit

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the recurrence relations between the polynomials and facilitate the analysis of their properties, as we will discuss next. Given the following result, several new identities can be derived.

Theorem 1 *Let F_r be the fundamental system of Fibonacci polynomials. Then, for $r \geq 2$, $1 \leq s \leq r-1$, and $n \geq 1$, we have:*

$$F^{(s)}_{n+r-1}(x) = det(R_{n,s}) = det \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & x & -1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & x & -1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \cdots & 0 \\ 1 & 1 & \cdots & 1 & \ddots & \ddots & 0 \\ 0 & 0 & 1 & \cdots & \ddots & x & -1 \\ 0 & \cdots & 0 & 1 & \cdots & 1 & x \end{pmatrix}_{n\times n}.$$

$$R_{n,s_{i1}} = \begin{cases} 1, & \textit{if } 1 \leq i \leq s, \\ 0, & \textit{otherwise.} \end{cases}$$

***K*eywords** Chebyshev Polynomials · Generalized Fibonacci polynomials · Fundamental system

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ICMASE 2025

DETERMINANTAL APPROACH APPLIED TO THE NON-HOMOGENEOUS $(q,k)-$FIBONACCI-STIRLING SEQUENCE

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ABSTRACT

The sequence $\{F_n\}_{n\geq 0}$ of Fibonacci numbers is defined recursively by the relation $F_n = F_{n-1} + F_{n-2}$ with $F_0 = F_1 = 1$. Among its interpretations, we can highlight that these numbers represent the count of compositions of the integer $n+1$ with no part equal to 1 (see [2], Sequence A000045). On the other hand, the Stirling numbers of the first kind $S(n,k),\ 0 \leq k \leq n$, determine the number of permutations in the symmetric group S_n that decompose into exactly k cycles. Formulas for $S(n,k)$ in terms of partitions of a positive integer, cyclic types of a permutation, and Vieta's formula can be found, respectively, in the works of [3, 4] and [5].

In this work, we are interested in the non-homogeneous recurrence relation with variable coefficients given by:

$$P_{n+2}^{(k)}(q) = p_{1,n}^{(k)}(q)P_{n+1}^{(k)}(q) + p_{2,n}^{(k)}(q)P_n^{(k)}(q) + S(n+2,k),\ 0 \leq k \leq n, \tag{1}$$

where $p_{1,n}^{(k)}(q) = 1 - q^2 + q^{2n+2k+1}$, $p_{2,n}^{(k)}(q) = q^2$ and with initial conditions $P_0^{(k)}(q) = 1$ and $P_1^{(k)}(q) = 1 + q^{2k+1}$. Note that if we consider only the homogeneous part of (1) with the initial conditions, when $q \to 1$, we obtain the sequence $\{F_n\}_{n\geq 0}$ of Fibonacci numbers. Therefore, the homogeneous part of (1) is a q-analog of the sequence F_n, as corroborated in [1].

Now, if we take the vector $Y_n^{(k)} = (P_n^{(k)}(q), P_{n+1}^{(k)}(q))^T$ and $X_n^{(k)} = (0, S(n+1,k))^T$, from the recursive process we find that Equation (1) can be represented in matrix form as

$$Y_{n+1}^{(k)} = T_q^{(k)}(n+1)Y_0^{(k)} + \sum_{r=1}^{n}\prod_{i=r}^{n} L_q^{(k)}(i)X_r^{(k)} + X_{n+1}^{(k)}, \tag{2}$$

where $T_q^{(k)}(n) := \prod_{h=0}^{*,n-1} L_q^{(k)}(h)$ is the second order transition matrix, $L_q^{(k)}(n)$ is the companion matrix associated to Equation (1) and $Y_0^{(k)}$ is the vector of initial conditions. Thus, in order to find explicit formulas for the entries of $T_q^{(k)}(n)$, we will consider the canonical solutions $\psi_n^{(0)}(q,k)$ and $\psi_n^{(1)}(q,k)$ of Equation (1), which are defined, for $s \in \{0,1\}$, as $\psi_n^{(s)}(q,k) := \det \Phi_{n-1}^{(s)}$, for $n \geq 2$ and, when $n = 0, 1$, we define $\psi_n^{(s)} = 1$ for $n = 1 - s$ and 0 otherwise. The matrix $\Phi_{n-1}^{(s)}$ is the tridiagonal matrix generated by the first $n-1$ rows and columns of the infinite matrix $\Phi^{(s)}$, given by:

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$$\Phi^{(s)} = \begin{pmatrix} p_{1+s,0}(q) & p_{2+s,1}(q) & & & \\ -1 & p_{1,1}(q) & p_{2,2}(q) & & \\ & -1 & p_{1,2}(q) & p_{2,3}(q) & \\ & & \ddots & \ddots & \ddots \end{pmatrix},$$

and with $p_{m,n}(q) = 0$ whenever $m > 2$. Then, from the previous data, we are able to show that the explicit determinantal expression for the sequence $\{P_n^{(k)}(q)\}_{n\geq 0}$ is given by:

$$P_{n+2}^{(k)}(q) = (1+q^{2k+1})\psi_{n+2}^{(0)}(q,k) + \psi_{n+2}^{(1)}(q,k) + \sum_{r=1}^{n+1} \psi_{n-r+2}^{(0,r)}(q,k)S(r+1,k), \tag{3}$$

for all $0 \leq k \leq n$. Thus, from the formulas for the homogeneous solution of (1) obtained in [1] and the formulas for $S(n,k)$ found in [3]-[5], we can establish an explicit combinatorial formula for the sequence $\{P_n^{(k)}(q)\}_{n\geq 0}$.

***K*eywords** Fibonacci sequence · Stirling numbers of the first kind · Recurrence relations · Determinantal approach

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ECONOMICAL MATHEMATICAL MODEL WITH M-SERIES LOCAL DERIVATIVE

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ABSTRACT

In this paper, we submit the price adjustment equation, which moves a substantial role in market equilibrium, by considering the generalized $\mathcal{M}$-derivative with the $\mathcal{M}$ series described as the limit for α-differentiable functions. The essential advantage of these models, as per their classical versions is that the derivative be arbitrary orders. For this aim, the method for linear ordinary differential equations on the generalized $\mathcal{M}$-derivative approach is utilized and some major results are acquired with this method. The price adjustment equation, which acts a crucial role in supplying market equilibrium, is resolved with this method when the agents expectancys are considering or disregarded [1-5].

***K*eywords** Price adjustment equation · Generalized $\mathcal{M}$-derivative · Mathematical model

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ICMASE 2025

LAGRANGIAN RELAXATION FOR LOT SIZING

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ABSTRACT

This study addresses the lot-sizing problem, motivated by practical challenges in Brazilian food manufacturing environments. The production setting comprises multiple production lines that share limited resources, such as labor and machinery. Due to the scarcity of these resources and operational constraints, not all lines can operate simultaneously. Each line is specialized and solely responsible for producing specific items. The mathematical model was designed by Medeiros, Soler, and Queiroz [3], based on formulations proposed by Haase [2], and Soler, Santos, and Akartunali [4]. To accurately model item perishability, the variable redefinition technique introduced by Eppen and Martin [1] was adopted, enabling a suitable representation of product shelf life within the planning horizon. This study aims to apply Lagrangian Relaxation and heuristic methods to the Lot-Sizing Problem with Perishable Products and Production Lines Sharing Scarce Resources. The main objective is to improve upon the results reported in the literature, particularly on large-scale instances that are often more computationally demanding. The implementation was carried out using the Python programming language, integrated with CPLEX, a high-performance optimization solver widely adopted in operations research.

During the computational experiments, a significant increase in solution time was observed as the instance size increased. To address this challenge, Lagrangian Relaxation was applied using the subgradient method, which enables the decomposition of the original problem into more manageable subproblems [5]. Also, feasibility and improvement heuristics were developed to generate good-quality solutions efficiently. These heuristics were designed to accelerate convergence and enhance solution quality by addressing infeasibilities and refining upper bounds. The computational results indicated that the proposed approach significantly reduced processing time and, in some instances, provided better-quality solutions than those obtained exclusively by CPLEX within the established time limit.

***K*eywords** Lot Sizing Problem · Lagrangian Relaxation · Subgradient Method · Heuristics

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FIRST STEP OF TRANSFORMATION OPERATORS VIA LOCAL M-DERIVATIVE

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ABSTRACT

In this paper, a transformation operator defined in the fractional derivative approach is presented, which is different from the generally known concept of transformation operator. This construction, which is developed with the local $\mathcal{M}$- derivative defined via the Mittag-Leffler function, offers an alternative approach to classical models. As an alternative to the limitations of classical derivatives, this local $\mathcal{M}$- derivative offers more flexible solutions, especially for fractional order differentiation. In this context, it is shown that the kernel function of the transformation operator defined in the paper corresponds to the solution of the given spectral problem under certain conditions. Mathematical tools such as the partial $\mathcal{M}$- derivative concept based on the local $\mathcal{M}$- derivative and integration by parts formula were used in the study. These mathematical tools contribute to the solution of both theoretical and applied problems and offer a more comprehensive approach compared to classical derivative methods. The theoretical results obtained are supported with visuals to provide a more tangible understanding of the results. As a result, this study shows how fractional derivatives can be used more flexibly and effectively as an alternative to traditional mathematical methods [1-5].

***K*eywords** Spectral theory · Local $\mathcal{M}$-derivative · Transformation operator

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GENERALIZED CONVEX B-METRIC SPACES AND ENRICHED RATIONAL TYPE CONTRACTIONS ON QUASI-BANACH SPACES

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ABSTRACT

Let X be a nonempty set and $T : X \to X$ be a self mapping. If there exists a point $x \in X$ such that $Tx = x$, then x is called a fixed point of T. Banach [1] gave the famous Banach contraction principle as follows:

"Let (X, d) be a complete metric space and T be a self-mapping on X. If there exists $\kappa \in [0, 1)$ such that $d(Tx, Ty) \leq \kappa d(x, y)$, for all $x, y \in X$. Then, T has a unique fixed point. Furthermore, the Picard iteration $\{x_n\}$ defined by $x_n = Tx_{n-1}$, for all $n \in \mathbb{N}$, converges to the fixed point of T."

Banach contraction principle has been generalized by many researchers by revising the topology of the space or the contractive condition of the mapping T. With Berinde's [2] introduction of enrichment concept on Banach spaces, fixed point results for enriched contraction mappings became popular for researchers. In [2], enriched contraction introduced as follows:

Let $(X, \|.\|)$ be a linear normed space. A mapping $T : X \to X$ is said to be an enriched contraction if there exists $b \in [0, \infty)$ and $\theta \in [0, b + 1)$ such that

$$\| b(x - y) + Tx - Ty \| \leq \theta \| x - y \| \text{ for all } x, y \in X. \tag{1}$$

Moreover, they proved some fixed point theorems for Banach spaces by using Krasnoselskij iteration given by

$$x_{n+1} = (1 - \lambda)x_n + \lambda Tx_n, n \in \mathbb{N} \tag{2}$$

B-metric space has been introduced by Bakhtin [3] and researchers obtained many fixed point results in b-metric spaces. In [4], convex b-metric space was introduced and some fixed point theorems were extended to this new concept. Quasi-norm, which is a very related concept with the b-metric spaces, was given by Hyers [7]. Recently, Berinde [5] proved some fixed point results for enriched contraction mapping in quasi-norm spaces which are the generalizations of many enrichment results. In this work, we introduce the enriched rational type contraction mapping which is an extension of the contraction mapping given in (1) and we present some fixed point theorems for enriched rational type contractions in quasi-norm spaces and we obtain an extension of Dass-Gupta rational type fixed point result given in [6] to the enrichment concept by using Krasnoselskij iteration procedure. Also, we examine weak enriched rational type contraction by Kirk's iteration. Morever, by introducing the notion of generalized convex b-metric space, we give some fixed point results in generalized convex b-metric spaces.

Keywords Fixed point · Enriched rational type contraction · Quasi-norm

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ICMASE 2025

PRINCIPAL BUNDLES WITH PRESCRIBED MONODROMY AND APPLICATIONS TO TOPOLOGICAL CONTROL SYSTEMS

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ABSTRACT

Principal bundles with a complex reductive structure group are geometric objects of great relevance in mathematical research and provide numerous applications [1]. The theory of principal bundles with prescribed monodromy, based on the theory of principal bundles to which an additional structure is included, represents a significant area of research at the intersection of differential geometry and algebraic topology, with applications in robotic navigation and control systems. The present work makes original contributions to this field, building upon established foundations in the geometric theory of bundles [2] and representation spaces [7]. Operating within the context of compact Riemann surfaces with punctures [11], this research develops a novel characterization of principal G-bundles for complex reductive Lie groups equipped with connections having prescribed monodromy around specified points. This approach extends classical results from Simpson [14] on moduli spaces of representations to new contexts with direct applications to engineering problems.

The core innovation of the present research lies in establishing a precise bijective correspondence between character varieties and moduli spaces of principal bundles with prescribed monodromy. Explicit dimensional formulas are provided that quantify how the topological complexity of the underlying surface, the structure of the Lie group, and the nature of the prescribed monodromy classes interact to determine the global geometry of the moduli space. These formulas reveal that, for a semisimple Lie group G, the dimension depends on both the genus of the surface and the dimensions of the conjugacy classes involved. This provides new insights into the geometric structure of these spaces beyond what was previously understood through the work of Hitchin [8] and Boalch [4], and further developed in recent research [5]. This study also offers a detailed analysis of the singularity structure of these moduli spaces, connecting them to bundles with non-trivial automorphism groups in a way that extends traditional approaches from geometric invariant theory [12].

The most significant contribution of this work is the development of a geometric framework for applying the theory of principal bundles with prescribed monodromy to control systems operating in topologically complex environments. Thus, by reformulating the navigation problem for robotic systems [13] as a question about connections on principal bundles over punctured surfaces, the research connects geometric structures on principal bundles and engineering challenges. This approach builds upon but substantially extends the geometric formulation of robot navigation introduced by Koditschek and Rimon [9], providing a more sophisticated treatment of the topological constraints that arise in obstacle avoidance problems. We demonstrate how the local geometry of the moduli space can be interpreted as a measure of the sensitivity of control strategies to perturbations, offering a precise way to quantify robustness that goes beyond the algorithmic approaches typically found in works like that of LaValle [10].

Another original contribution is the characterization of when two control strategies can be continuously deformed into one another while maintaining obstacle avoidance constraints. By identifying

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the obstruction to such deformations as an element of a specific cohomology group, the work establishes a topological classification of control strategies that provides new insights into the fundamental limitations of control system design. This approach connects to recent developments in applied algebraic topology for dynamical systems [3].

The methodology used in this research builds bridges between the algebraic geometric theory of principal bundles and character varieties and engineering concerns in control theory. This is in line with certain emerging approaches in computational topology for data analysis [6] but with a focus on continuous dynamical systems rather than discrete data structures.

***K*eywords** Principal G-bundles · Prescribed monodromy · Character varieties · Moduli spaces · Topological control theory

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LIMIT DISTRIBUTIONS OF EXTREME ORDER STATISTICS FOR THE GENERALIZED PARETO DISTRIBUTION

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ABSTRACT

The generalized Pareto distribution (GPD) is a useful probability model in the case of exceedance above a high threshold. It can be reduced to uniform, exponential, and Pareto distributions according to different values of its parameters. The GPD is widely used in various fields, including actuarial sciences, economics, engineering, and numerous other disciplines ([1, 2]). Considering the importance of extreme values in applications, we study the necessary and sufficient conditions of the domain of minimal and maximal attractions for the GPD. Domains of the minimal and maximal attractions are obtained based on the various cases of the shape parameters. Necessary and sufficient conditions are checked for convergence for minimum and maximum order statistics following the result summarized by Fisher and Tippett [3]. The asymptotic results are compared with exact values by variability measures.

***K*eywords** Asymptotic distribution · Domain of attraction · Extreme value theory · Order statistics · Generalized Pareto distribution

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ON GAUSSIAN GENERALIZED EDOUARD NUMBERS

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ABSTRACT

In the literature, many studies have been devoted to Gaussian numbers whose components are taken from special integer sequences, such as Fibonacci, Lucas, Pell, and Jacobsthal, (see [1, 2, 3, 4, 5, 6, 8], and references therein). Gaussian numbers have an algebraic and geometric structure, but especially when they are elements of recurring sequences, such as Horadam's and Leonardo's numbers, they define complex generalizations of generalized Fibonacci numbers.

This numbering system, the Gaussian numbers, was introduced by Horadam in [4]. Consider the field of complex numbers, denoted by $(\mathbb{C}, +, \cdot)$. The set of complex numbers is defined as $\mathbb{C} = \{a + bi \mid a,\ b \in \mathbb{R} \text{ and } i^2 = -1\}$, where i is the complex unit. It was established that $(\mathbb{C}, +)$ and $(\mathbb{C}, \cdot)$ are abelian groups. A Gaussian number is a complex number $z = a + bi$, where a and b are integers, see [2].

Following the definition given in [5], the Gaussian Fibonacci sequence of numbers $\{GF_n\}_{n\geq 0}$ and the Gaussian Lucas sequence of numbers $\{GL_n\}_{n\geq 0}$ are defined by the recurrence relation $GF_{n+1} = GF_n + GF_{n-1}i$, where $GF_0 = i, GF_1 = 1$, and $GL_{n+1} = GL_n + GL_{n-1}i$, where $GL_0 = 2 - i, GL_1 = 1 + 2i$, respectively. Some identities involving Gaussian Fibonacci sequences are established in [2].

In this article, we introduce the generalized Gaussian Edouard numbers $\{GW_n\}_{n\geq 0}$ defined by the recurrence relation

$$GW_m = 7GW_{m-1} - 7GW_{m-2} + GW_{m-3},$$

with arbitrary initial conditions GW_0, GW_1, GW_2. We establish the recurrence relation, generating function and the Binet formula for this new sequence of numbers.

As particular cases, when $GW_0 = 0, GW_1 = 1, GW_2 = 7 - i$, we have the Gaussian Edouard numbers $\{GE_n\}_{n\geq 0}$, and by taking $GW_0 = 3, GW_1 = 7 - 3i, GW_2 = 35 - 7i$, we get the Gaussian Edouard–Lucas numbers $\{GK_n\}_{n\geq 0}$. The relations of these new particular sequences with the Gaussian Balancing and Gaussian Lucas-Balancing numbers (see [8]) are explored, and some new identities are provided.

***K*eywords** Generalized Edouard numbers · Generating function · Binet's formula · Identities

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ANALYSIS OF A GENERALIZED ROBIN–STEKLOV EIGENVALUE PROBLEM WITH THE (P,Q)-LAPLACIAN

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ABSTRACT

In this paper, we investigate an eigenvalue problem driven by the $(p,q)-$Laplacian, involving positive potentials and parametric boundary conditions. By employing the Nehari manifold method along with variational techniques, we prove the existence of a nontrivial open interval $I \subseteq \mathbb{R}$ such that every $\lambda \in I$ is an eigenvalue of the problem.

***K*eywords** Eigenvalues · $(p,q)-$ Laplacian · parametric boundary condition · Nehari manifold

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TRANSMISSION EIGENVALUE PROBLEMS WITH NEUMANN–ROBIN BOUNDARY CONDITIONS INVOLVING THE P- AND Q-LAPLACIAN

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ABSTRACT

This work investigates a transmission eigenvalue problem with Neumann–Robin boundary conditions. The existence of an infinite sequence of eigenvalues is established using Lusternik–Schnirelmann theory, applied in the setting of C1- Banach manifolds and based on the Krasnoselskii genus. This variational approach enables us to prove the existence of an unbounded sequence of eigenvalues associated with the problem.

***K*eywords** Nonlinear transmission problem · p-Laplacian · Sobolev spaces · Krasnoselskii genus · Lusternik–Schnirelmann theory.

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ICMASE 2025

APPLIED MATHEMATICAL ANALYSIS OF PERFECT DOMINATION IN OTTOMAR GRAPHS

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ABSTRACT

If $N(V) \cup S \neq \emptyset$, then a set $S \subseteq V(G)$ is a dominate set in G. For each vertex $v \in V - S$. The dominance number of G, represented as $\gamma(G)$, is the minimal cardinality of all dominating sets in the graph. Now what do we mean by perfect domination? Suppose G is graph, simple and connected. If exactly one element of S dominates $x \in V(G)\backslash S$, then the dominating set $S \subseteq V(G)$ is termed to as the perfect dominating set of G. The minimum cardinality of a perfect dominating set of G is a perfect domination number, denoted as $\gamma_p(G)$. The Ottomar graph, represented in this study as $O_{(n,m)}$, is the graph C_n, $n \in \mathbb{Z}^+$, $n \geq 3$, with a vertex connected to a vertex of C_m, $m \in \mathbb{Z}^+$, $n \geq 3$ by a path P_2. C_m is called foot (plural: feet), while C_n is called the heart. In this work, dominating, inverse dominating, and the perfect domination number of an Ottomar graph are studied. The minimal cardinality of a perfect dominating number of an Ottomar graph is represented by $\gamma_p(O_{n,m})$. We also study the invers perfect dominate of Ottomar $\gamma_p^{-1}(O_{n,m})$, then the perfect, dominating set of $\gamma_p^{-1}(O_{n,m})$ cardinality is called $\gamma_p^{-1}(O_{n,m})$-set.

***K*eywords** Ottomar graph · Perfect domination · Domination · Inverse · Inverse domination · Inverse perfect domination

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ISOMONODROMIC CONNECTIONS ON PRINCIPAL BUNDLES AND DYNAMICAL SYSTEMS

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ABSTRACT

A principal G-bundle P over a manifold M consists of a total space P, a base space M, a projection $\pi : P \to M$, and a free right action of a Lie group G on P that preserves the fibers of π. These geometric objects provide a natural framework for studying systems with internal symmetries [1]. In this context, connections on principal bundles represent geometric structures that enable horizontal lifting of curves from the base space to the total space, facilitating the study of parallel transport and holonomy. Logarithmic connections, a specific class of connections with simple poles at designated points, emerge in the study of differential equations with regular singular points and provide a rich geometric structure for analyzing control systems. The monodromy of a logarithmic connection, which measures the holonomy around loops encircling the singular points, serves as a topological invariant that characterizes the global behavior of the connection.

Isomonodromic deformations constitute particularly interesting families of connections, where the monodromy representation remains constant as the connection varies. These deformations are governed by the Schlesinger equations, a system of non-linear partial differential equations that describe how the residues of the connection must evolve to preserve the monodromy [5].

Control systems on manifolds represent a fundamental area of study in control theory, with applications ranging from robotics to aerospace engineering. The geometric approach to control theory, pioneered by [4], applies differential and algebraic geometry for understanding controllability, stability, and optimality of trajectories. When the configuration space possesses non-trivial topology, as is often the case for mechanical systems with constraints or symmetries [2], the geometric perspective becomes essential for a comprehensive analysis.

The main contribution of this work is the establishment of a precise correspondence between isomonodromic deformations of logarithmic connections on principal G-bundles and control systems on Riemann surfaces. This correspondence illuminates the geometric structures underlying controllability and optimality of trajectories, leading to several key insights. First, the monodromy-curvature correspondence serves to show how the curvature of a pulled-back connection relates to the residues of the original logarithmic connection. This result provides a bridge between the local differential structure of the connection and its global topological properties, allowing for a more comprehensive understanding of the system's behavior. Second, the structure of isomonodromic deformations is characterized through a non-linear partial differential equation satisfied by the map from the Riemann surface to the homogeneous space G/H, where H is a maximal compact subgroup of G. This equation encodes the evolution of the connection as the singular points move, providing a concrete description of how the geometry changes during the deformation.

The central theorem establishes a precise relationship between isomonodromic families of logarithmic connections and G-equivariant control systems on the total space of the principal bundle. This theorem demonstrates that the controllability of the system is equivalent to the condition that the

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residues of the connection generate the full Lie algebra $\mathfrak{g}$ under the Lie bracket operation. Furthermore, the trajectories of the control system project to geodesics on the Riemann surface with respect to a metric determined by the monodromy data, and the optimal control functions minimizing a quadratic cost functional correspond precisely to isomonodromic deformations of the connection. As explored by [3] in a work on nonholonomic mechanics, systems with constraints require geometric techniques for effective control. Thus, the approach developed here provides a systematic method for designing robust controllers that account for the geometric complexities of the task space, ensuring optimal performance even in challenging environments.

This geometric framework is extended to control systems with non-holonomic constraints, establishing a representation in terms of families of logarithmic connections where the singular points encode the geometric properties of the constraint distribution. This extension provides a tool for analyzing systems with velocity constraints, such as wheeled robots or underwater vehicles. This has implications beyond robotic control, extending to areas such as quantum control theory, where the geometric phases become crucial for understanding the evolution of quantum systems. The connection with integrable systems, emphasized by [6], suggests potential applications to machine learning algorithms for control on manifolds, where the preservation of geometric structures could lead to more efficient and robust learning processes.

Finally, a detailed computational example for $\mathrm{SU}(2)$ connections over a hyperbolic Riemann surface of genus 2 is provided, illustrating the applicability of the geometric framework. The example considers the specific case of controlling the orientation of a robot moving on a curved surface, demonstrating how the abstract theory translates into concrete control strategies. It begins with the choice of three singular points in the upper half-plane and the definition of residues that form a basis for the Lie algebra $\mathfrak{su}(2)$, ensuring controllability of the system. The computation of the map from the Riemann surface to the homogeneous space $\mathrm{SU}(2)/\mathrm{U}(1)$ and the associated metric provides insight into the geometric structure induced by the connection. The geodesics with respect to this metric correspond to optimal trajectories for the control system, minimizing a quadratic cost functional related to the energy expenditure of the control inputs. The resulting control strategy guides the robot along a nearly vertical path in the upper half-plane while optimally adjusting its orientation, with robustness to perturbations ensured by the isomonodromic property of the connection. The robustness of this control strategy stems from the isomonodromic property: small perturbations of the path preserve the monodromy representation, ensuring that the system can recover from disturbances without significant deviations from the optimal trajectory. This property is particularly remarkable for robotic systems operating in uncertain environments or on surfaces with varying curvature, as discussed by Marsden and Ratiu [7].

***K*eywords** Isomonodromic connections · Principal bundles · Dynamical systems · Geometric control theory

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ICMASE 2025

A Solution to the Fourth-Order Nonlinear Problem with Symmetric Boundary Conditions Using the Banach Fixed Point Theorem and Green's Function

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Abstract

In this study, a fourth-order nonlinear differential boundary value problem is considered. The problem has a structure in which the solution function depends on both itself and its first derivative, and it is defined by symmetric mixed boundary conditions. First, the Green's function appropriate to the problem is constructed. Then, the existence, uniqueness, and iterative convergence of the solution are proven using the Picard iteration method in conjunction with the Banach fixed point theorem. Theoretical results are supported by a numerical example, and it is observed that the iteration process converges rapidly..

***K*eywords** Banach Contraction Principle · Green Function · Fouth Order BVP

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ICMASE 2025

GOLDEN RATIO-BASED TWO-STEP SUBGRADIENT EXTRAGRADIENT METHODS FOR VARIATIONAL INEQUALITIES: CONVERGENCE ANALYSIS AND APPLICATIONS IN SIGNAL AND IMAGE PROCESSING

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ABSTRACT

We propose two novel inertial algorithms for solving variational inequality problems in Hilbert spaces. The first method integrates the golden ratio technique into a two-step subgradient extragradient framework, achieving weak convergence under monotonicity and R-linear convergence under strong monotonicity. The second algorithm modifies this approach to ensure strong convergence of the generated sequence. Both methods employ adaptive step sizes, eliminating the need for Lipschitz constant estimation or line search. Numerical experiments demonstrate their competitive performance compared to existing techniques. Additionally, applications in signal recovery and image restoration highlight their practical superiority over state-of-the-art methods.

Keywords Golden Ratio Technique · Inertial Two-subgradient Extragradient Method · Variational Inequality Problems · Monotone VIPs · Self-adaptive Step Sizes · Image Restoration · Optimization

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A BRIEF STUDY OF THE ONE PARAMETER LICHTENBERG NUMBERS

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ABSTRACT

In the 17th century, Georg Christoph Lichtenberg (1742–1799), as a result of his research on Chinese rings, introduced the following recurrence

$$Li_n = 2^n - 1 - Li_{n-1}\,, \tag{1}$$

for all integer $n \geq 1$, and with initial values $Li_0 = 0$ (see sequence $A000975$ in [8]). Hinz [5] and Cerda-Morales [3] called the sequence of numbers $\{Li_n\}_{n\geq o}$, the *Lichtenberg numbers* and established the nonhomogeneous recurrence relation

$$Li_n = Li_{n-1} + 2Li_{n-2} + 1\,, \tag{2}$$

for all integer $n \geq 2$, with initial terms $Li_0 = 0$ and $Li_1 = 1$. Equation (2) defines the sequence of *Ernst* numbers introduced by Soykan in cite soykan2022. For historical reasons, we will use the name Lichtenberg numbers, according to the authors Hinz [5], Stockmeyer [6], and also Heeffer and Hinz [7].

The Lichtenberg numbers are interesting because they are closely related to the well-known Jacobsthal numbers. The sequence of Jacobsthal numbers is denoted by $\{J_n\}_{n\geq 0}$ and defined by the recurrence relation $J_n = J_{n-1} + 2J_{n-2}$ with initial values $J_0 = 0$ and $J_1 = 1$ (see sequence $A001045$ in [8]). In fact, Cerda-Morales [3] determined the identity

$$Li_n = \frac{J_{n+2} - 1}{2}. \tag{3}$$

On the other hand, we highlight the work of Anatassov in [1, 2], who introduced a generalization of s-Jacobsthal numbers, as follows

$$J_{(s,n)} = \frac{s^n - (-1)^n}{s+1}, \tag{4}$$

for every integer $n \geq 2$, arbitrary real number s, and initial values $J_{(s,0)} = 0$ and $J_{(s,1)} = 1$. In addition, we can define the s-Jacobsthal-Lucas numbers as follows

$$j_{(s,n)} = s^n + (-1)^n, \tag{5}$$

for $n \geq 2$, and with $j_{(s,0)} = 2$ and $j_{(s,1)} = 1$.

Motivated by identities (3) and (4), our goal is to introducethe s-Lichtenberg and s-Lichtenberg-Lucas numbers for some real s, and study some properties of this new sequence of numbers. More

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precisely, we give a recurrence for the s-Lichtenberg and s-Lichtenberg-Lucas numbers by using, respectively, the s-Jacobsthal and s-Jacobsthal-Lucas numbers. We show a relation between the s-Lichtenberg, s-Lichtenberg-Lucas, s-Jacobsthal and s-Jacobsthal-Lucas numbers and explore the connection between the s-Lichtenberg numbers, the Lichtenberg numbers, and the Jacobsthal numbers establishing some properties related to the s-Lichtenberg and s-Lichtenberg-Lucas numbers. In addition, Binet's formulas are obtained. Finally, we examine some properties of these new sequences, including the classical identities.

***K*eywords** Lichtenberg numbers · Jacobsthal numbers · Binet's formula · Identities

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ICMASE 2025

APPROXIMATION BY CHLODOWSKY-TYPE OF SZÁSZ OPERATORS INCLUDING THE APPELL POLYNOMIALS OF CLASS $\mathbb{A}^{(2)}$

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ABSTRACT

A Chlodowsky variation of generalized Szász type operators and a novel sequence of operators, containing the Appell polynomials of class $\mathbb{A}^{(2)}$, are the subjects of this study. Approximation properties and convergence results are given by using different types of modulus of continuity with the help of Steklov function. A weighted space of functions constructed on $[0, \infty)$ is used to study the convergence features of these operators. Theoretical conclusions are demonstrated by using the Gould-Hopper and Hermite polynomials.

***K*eywords** Appell polynomials · weighted space · rate of convergence · voronovskaya-type theorem

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NON-LINEAR INAR(1) MODELS

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ABSTRACT

Integer-valued autoregressive models play a significant role in the study of count time series. These models are composed of the survival and the innovation component. In this study we discuss models with a non-linear structure, where the non-linearity is achieved by modifying the survival component. Namely, the idea is to introduce some additional processes into the survival component in order to control the influence of the previous values on the current one. Although autoregression is inherent in the series, we aim to enhance or diminish its effect by incorporating this additional process. Its affect on the model is with a certain probability. The methods for the parameters estimation are presented and their efficiency is investigated on simulated data sets. The application of the model is discussed through some real data sets.

Keywords INAR · Binomial thinning operator · Negative binomial thinning operator

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ON COMPLEX SEQUENCE OF VIETORIS' NUMBERS AND THEIR FINITE SUMS

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ABSTRACT

Special integers sequences and their properties have been the center of attention for many authors in number theory. Their polynomial sequences, hybrid extensions, hypercomplex applications, partial sums and congruences, quaternionic sequences and etc. are the focus of many studies. In this study, motivated by a rational sequence known as Vietoris' numbers, we investigate the complex sequence of Vietoris' numbers and their properties. Some recurrence-like relations and norm identities of this sequence are established. Moreover, we deduced some finite summations involving the terms of complex Vietoris' numbers.

***K*eywords** Central binomial coefficients · Complex numbers · Vietoris' sequence · Finite sums

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THE FUNDAMENTAL SYSTEM OF PELL POLYNOMIALS AND THE COMPANION MATRIX

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ABSTRACT

The well-known Pell polynomial sequence, denoted by $\{P_n(x)\}_{n\geq 0}$ is defined by the initial conditions $P_0(x) = 0$, $P_1(x) = 1$, and the recurrence relation $P_{n+1}(x) = 2xP_n(x) + P_{n-1}(x)$, for $n \geq 1$, as presented in the literature (see, for example, [1], [4],[5]). Generalizations of this sequence can be found in the literature (see, for instance, [3], [8], [5]), along with numerous applications in algebra, analysis, combinatorics, and matrix theory. In [8], the Pell fundamental system is defined by the recurrence relation $P_n^{(s)} = P_{n-1}^{(s)} + P_{n-2}^{(s)} + \cdots + P_{n-r}^{(s)}, \quad n \geq r$, with initial conditions given by $P_n^{(s)} = \delta_{s-1,n}, \quad 0 \leq n \leq r-1$. Thus, considering these results, in this work, we focus on a generalization given by the following linear difference equation of order $r \geq 2$:

$$P_n(x) = 2xP_{n-1}(x) + \sum_{i=1}^{r-1} P_{n-i-1}(x), \quad \forall n \geq r, \tag{1}$$

with the initial conditions $P_0(x), P_1(x), \ldots, P_{r-1}(x)$. We call the class of polynomials given by Equation (1) the Pell polynomials. Our goal is to study Equation (1) via Pell fundamental system and the companion matrix associated. Considering the properties of the fundamental system, that is, the relationship between the sequences of Pell polynomials, as well as the connection of this family of sequences with the power of the associated companion matrix, several new results are derived, that is, new results of generalizations of properties already known for Pell polynomials or even Pell numbers.

Based on the connection between Markov chains and sequences of linear recurrence relations presented in [6] and results presented in [2], several combinatorial results for generalized Pell polynomials are derived.

In [6] we have that for $n > m \geq r$, the number $\rho(n, m)$ is a probability, given by:

$$\rho(n,m) = \sum_{k_0+2k_1+\cdots+rk_{r-1}=n-m} \frac{(k_0+\cdots+k_{r-1})!}{k_0!k_1!\ldots k_{r-1}!} a_0^{k_0} \ldots a_r^{k_{r-1}}. \tag{2}$$

In [2] we have that the entries of the power of an associated companion matrix can be written as:

$$c_{ij}^{(n)} = \sum_{k_1+2k_1+\ldots+mk_m=n-i+j} \frac{k_j+\cdots+k_m}{k_1+\cdots+k_m} \frac{(k_1+\cdots+k_m)!}{k_1!k_1!\ldots k_m!} a_1^{k_1} \ldots a_m^{k_m}, \tag{3}$$

Thus, considering these results, we can write the entries of the companion matrix associated with the Pell polynomials and obtain the following expression.

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$$P_n^{(r)}(x) = \rho(n+1,r) = \sum_{k_0+2k_1+\cdots+rk_{r-1}=n+1-r} \frac{(k_0+\cdots+k_{r-1})!}{k_0!k_1!\ldots k_{r-1}!}(2x)^{k_0}, \quad n \geq r, \quad (4)$$

where $\rho(1,r) = \cdots = \rho(r-1,r) = 0$ and $\rho(r,r) = 1$.

***K*eywords** Generalized Pell polynomials · Fundamental system · combinatorial identities · Companion Matrix

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A BI-PERIODIC q-ANALOGUE FOR THE POWERS-OF-TWO SEQUENCE: COMBINATORIAL AND DETERMINANTAL APPROACHES

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ABSTRACT

A very important sequence in the area of discrete mathematics as well as in Number Theory is given by the recurrence relation $P_n = 2P_{n-1}$, with $P_0 = 1$. It refers to the powers of the integer 2, since $P_n = 2^n$ for all $n \geq 0$. This sequence is closely linked to Set Theory, as it represents the number of subsets of a set with n elements ([3], p. 8). By the other hand, this sequence represents the number of compositions of the integer n (see [4], Eq. (39)). Furthermore, several other interpretations are associated with the sequence $\{P_n\}_{n\geq 0}$, as can be seen in [7].

However, in this work, we focus on a q-analogue of the sequence $\{P_n\}_{n\geq 0}$, which is a polynomial in the indeterminate q that generalizes the sequence under consideration. That way, consider the following bi-periodic sequence with variable coefficients,

$$P_n(q) = \begin{cases} (1+q)P_{n-1}(q) + (q^{2n} - q)P_{n-2}(q), & \text{if } n \text{ is even} \\ (1+q)P_{n-1}(q) + (q^{2n-1} - q)P_{n-2}(q), & \text{if } n \text{ is odd} \end{cases}, \tag{1}$$

and with initial conditions $P_0(q) = 1$ and $P_1(q) = 1 + q$. Clearly, when $q = 1$, we have $P_n(1) = P_n = 2^n$, regardless of the parity of the sequence index. Although Equation (1) yields a bi-periodic polynomial sequence, we can represent $\{P_n(q)\}_{n\geq 0}$ as

$$P_n(q) = (1+q)P_{n-1}(q) + (q^{2n-\xi_n} - q)P_{n-2}(q),$$

where $\xi_n = \dfrac{1-(-1)^n}{2} = \begin{cases} 0, & \text{if } n \text{ is even} \\ 1, & \text{if } n \text{ is odd} \end{cases}$ is the parity function defined by [2]. Now, observe that, if we substract $P_{n+1}(q) - P_n(q)$ when n is even and odd, respectively, we obtain

$$P_{n+1}(q) = (2+q)P_n(q) - (1 + 2q - q^{2n+1})P_{n-1}(q) + (q - q^{2n})P_{n-2}(q), \tag{2}$$
$$P_{n+1}(q) = (2+q)P_n(q) - (1 + 2q - q^{2n+2})P_{n-1}(q) + (q - q^{2n-1})P_{n-2}(q). \tag{3}$$

Thus, Equations (2) and (3) allows us to establish that the sequence $\{P_n(q)\}_{n\geq 0}$ satisfy the following property

$$P_{n+1}(q) = (2+q)P_n(q) - (1 + 2q - q^{2n-\xi_{n+1}+2})P_{n-1}(q) + (q - q^{2n-\xi_n})P_{n-2}(q),\ n \geq 2. \tag{4}$$

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Consider the vector $Y_n = (P_n(q), P_{n+1}(q))^T$. Then, from the recursive process we find that Equation (1) can be represented in matrix form as $Y_{n+1} = T_q(n+1)Y_0$, where $T_q(n) := \prod_{h=0}^{*,n-1} L_q(h)$ is the second order transition matrix, $L_q(n)$ is the companion matrix associated to Equation (1) and Y_0 is the vector of initial conditions. Thus, in order to find explicit formulas for the entries of $T_q(n)$, we will consider the canonical solutions $\psi_n^{(0)}(q)$ and $\psi_n^{(1)}(q)$ of Equation (1), which are defined, for $s \in \{0,1\}$, as $\psi_n^{(s)}(q) := \det \Phi_{n-1}^{(s)}$, for $n \geq 2$ and, when $n = 0, 1$, we define $\psi_n^{(s)}(q) = 1$ for $n = 1 - s$ and 0 otherwise.

The matrix $\Phi_{n-1}^{(s)}$ is the tridiagonal matrix generated by the first $n-1$ rows and columns of the infinite matrix $\Phi^{(s)}$, namely

$$\Phi^{(s)} = \begin{pmatrix} p_{1+s,0}(q) & p_{2+s,1}(q) & & & \\ -1 & p_{1,1}(q) & p_{2,2}(q) & & \\ & -1 & p_{1,2}(q) & p_{2,3}(q) & \\ & & \ddots & \ddots & \ddots \end{pmatrix},$$

where $p_{1,n}(q) = 1+q$, $p_{2,n}(q) = q^{2n-\xi_n+4} - q$ and with $p_{m,n}(q) = 0$ whenever $m > 2$. Then, from the previous data, we are able to show that the explicit determinantal expression for the sequence $\{P_n(q)\}_{n\geq 0}$ is given by

$$P_n(q) = (1+q)\psi_n^{(0)}(q) + \psi_n^{(1)}(q), \tag{5}$$

for all $n \geq 0$. Thus, from the formulas for the determinants $\{\psi_n^{(s)}(q)\}_{n\geq 0}$ obtained in [1], we can establish an explicit formula for Equation (5) in terms of nested sums. By the other hand, the sequence $\{P_n(q)\}_{n\geq 0}$ offer interesting combinatorial interpretations for some series-product identities, listed by Slater in [6] and also by Santos in [5].

***K*eywords** Powers-of-two sequence · Recurrence relations · Variable coefficients · Bi-periodic sequences · Determinantal approach

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GEODESICS ON SPACE–LIKE HELICOIDAL SURFACES IN MINKOWSKI 3–SPACE

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ABSTRACT

In this talk, the equations of geodesics on the space–like helicoidal surfaces in the forms of integral will be found by using Euler–Lagrange equations in Minkowski 3–space. Since the all formulas are rather cumbersome for calculations, Simpson's formulas to calculate certain integrals will be used.

Acknowledgements The authors gratefully thank the financial support provided by the Scientific Research Projects Unit of Yozgat Bozok University (Grant No: FHD–2025–1667).

***K*eywords** Geodesics · Euler–Lagrange equation · Helicoidal surface · Minkowski 3–space

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ON GENERALISED LEONARDO NUMBERS

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ABSTRACT

The Leonardo sequence (Le_n) is defined by the inhomogeneous recurrence relation

$$\mathrm{Le}_n = \mathrm{Le}_{n-1} + \mathrm{Le}_{n-2} + 1\,, \quad \text{for } n \geqslant 2, \tag{1}$$

with initial conditions

$$\mathrm{Le}_0 = \mathrm{Le}_1 = 1\,. \tag{2}$$

Alternatively, it can be defined by the homogeneous recurrence relation

$$\mathrm{Le}_n = 2\,\mathrm{Le}_{n-1} - \mathrm{Le}_{n-3}\,, \quad \text{for } n \geqslant 3, \tag{3}$$

where in this case $\mathrm{Le}_0 = \mathrm{Le}_1 = 1$ and $\mathrm{Le}_2 = 3$ are the initial conditions. As with so many mathematical concepts, it is not always easy to establish when Leonardo numbers were first defined. This sequence has a *sui generis* history: first we found its extensions and more recently its interest has been reborn through very particular cases of them.

This talk covers some of the history of Leonardo numbers. We retrieve some of the most recent results on this sequence, as well as some relevant historical interconnections. Next, we present a matricial approach based on the determinant of certain Hessenberg matrices to finding the generating function of homogeneous recurrence relations, applying it in some examples after converting the inhomogeneous initial to the pertinent homogeneous form. In the end, we also provide some conjectures and open problems for some of its extensions involving the modular periodicity.

***K*eywords** Leonardo sequence · recurrence relations · Hessenberg matrices · determinant

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ICMASE 2025

CHARACTER-THEORETIC APPROACHES TO DIFFERENCE SETS

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ABSTRACT

Let G be a finite abelian group of order v, and let $D \subseteq G$ with $|D| = k$. D is called a (v, k, λ)-difference set if the list of differences $\{d - d' \mid d, d' \in D, d \neq d'\}$ covers every nonzero element of G exactly λ times. Character theory provides a powerful algebraic method for investigating such sets, particularly through the evaluation of group character sums.

In this study, we explore the classical character-theoretic condition which states that $D \subseteq G$ is a (v, k, λ)-difference set if and only if for every non-principal character χ of G,

$$\left| \sum_{d \in D} \chi(d) \right|^2 = k - \lambda v.$$

Using this condition, we investigate known difference sets such as the Singer difference sets and difference sets arising from cyclotomic classes in finite fields $\mathbb{F}_q$, where $q \equiv 1 \pmod{n}$.

Furthermore, we apply group ring techniques to represent subsets of G as formal sums in $\mathbb{Z}[G]$, and express the difference set condition as:

$$DD^{(-1)} = k \cdot 1_G + \lambda(G - 1_G),$$

where $D^{(-1)} = \{d^{-1} \mid d \in D\}$ and 1_G is the identity of G. This formulation connects combinatorial properties to algebraic identities and facilitates computer-aided verification.

Our work emphasizes how character sums not only simplify verification of known constructions but also guide the discovery of new families of difference sets. The results suggest further applications in combinatorial design theory, coding theory, and finite geometry.

***K*eywords** character theory · difference sets · group rings

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ICMASE 2025

THE HYBRID DECISION-MAKING MODEL WITH THE NEW FERMATEAN FUZZY ENTROPY MEASURE

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ABSTRACT

This paper proposes a new Fermatean fuzzy entropy for Fermatean fuzzy sets. Fermatean fuzzy entropy determines the quantity of information in the Fermatean fuzzy set. Thus, the proposed entropy provides a new, flexible, valuable tool in complex multi-criteria problems where uncertain data and inaccurate information are considered. In this study, an integrated multi-criteria decision-making method consisting of entropy, the "Pivot Pairwise Relative Criteria Importance Assessment", and "Measurement of Alternatives and Ranking according to Compromise Solution" methods based on a Fermatean fuzzy set is presented.

***K*eywords** Entropy · Fermatean fuzzy set · decision-making

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GRAPH-THEORETIC INSIGHTS INTO THE MOORE-PENROSE INVERSE OF RECTANGULAR FIBONACCI MATRICES WITH CRYPTOLOGICAL APPLICATIONS

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ABSTRACT

Cryptology, one of the key components of modern information security, continues to evolve in response to vulnerabilities in traditional encryption methods. In addition to classical approaches based on algebraic structures or number theoretic principles, the use of interdisciplinary mathematical methods such as graph theory offers innovative possibilities to improve cryptographic security.
This research introduces a novel cryptographic framework that merges the inherent structural characteristics of graph theory with Fibonacci-based matrices and their Moore-Penrose generalized inverses (pseudoinverses), offering a novel approach that exhibits robust resilience against conventional cryptanalysis techniques. The study focuses on strengthening the role of graph theory in cryptography by incorporating concepts such as complete graphs, weighted graphs, and cycles in graphs. Although there are various studies in the literature on the use of the Fibonacci sequence as a cryptographic key, the approach presented here, which combines the Moore-Penrose inverse of the Fibonacci matrix with Graph Theory, introduces a relatively new perspective that has not yet been widely explored in existing research. Thus, a hybrid model is created that incorporates the strengths of both linear algebra and graph theory into the cryptographic process simultaneously. From an information theoretic and computational security perspective, this model stands out by providing a different perspective and an additional layer of protection compared to existing methods.

***K*eywords** Graph Theory · Cryptography · Fibonacci Numbers · Fibonacci Q- matrix · The Moore-Penrose Generalized Inverse

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LUCAS WAVELET SOLUTION OF MATHEMATICAL MODEL FOR ALCOHOL ADDICTION

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ABSTRACT

This study focuses on the mathematical model of alcohol addiction with the influence of public and private addiction treatment centers. The spread of alcohol consumption behaviour is given as an epidemic model through the potential drinkers, moderate drinkers, heavy drinkers and recovered class. In this model, the recovery is achieved by the individual quitting alcohol consumption and stopping by getting treatment. The influence on heavy drinkers of well-equipped advanced interventions and non-advanced interventions with low quality services are analyzed through rich heavy drinkers and poor heavy drinkers. So, the model describes the interaction between six compartments that are potential drinkers (P), moderate drinkers (M), heavy drinkers (H), rich heavy drinkers (T^r), poor heavy drinkers (T^p) and quitters (Q). The mathematical model is a system of six nonlinear ordinary differential equations. The literature of this system lacks the analytic solution. So, in order to obtain approximate solutions of the system, the Lucas wavelet method is used to solve the equations numerically. Furthermore, we solve the model with different values of the parameters to investigate the effects of the problem parameters on the spread of Alcohol consumption behaviour. The Lucas wavelet method is easy to implement, since linearization of the nonlinear terms and discretization of the time interval are not required. This provides a computational advantage for the method. In order to examine the efficiency, the obtained solutions are compared with the fourth order Runge Kutta method. The accuracy of the solutions is checked via residual error analysis, and the results are given in graphics.

Keywords Alcohol Addiction · Mathematical Model · Treatment · Nonlinear System · Lucas Wavelets

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ICMASE 2025

QUANTUM CALCULUS AND FACTORIZATION OF FIBONACCI NUMBERS

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ABSTRACT

Factorization of q-deformed numbers $[n]_q$ and their generalizations in the set of the deformed prime numbers with base parameters as powers of q is established. We start from simple identity

$$(1+q)(1+q^2)(1+q^4)...(1+q^{2^n}) = \frac{1-q^{2^{n+1}}}{1-q},$$

which can be interpreted it as factorization of q-numbers

$$\prod_{k=0}^{n-1} [2]_{q^{2^k}} = [2^n]_q$$

and generalize it for arbitrary number p. For $|q|<1$ it includes an infinite product factorization of $[\infty]_q$ in q-deformed primes

$$\prod_{k=0}^{\infty}(1+q^{3^k}+q^{2\cdot 3^k}) = \frac{1}{1-q} = \prod_{k=0}^{\infty}[3]_{q^{3^k}}$$

For quantum calculus with two basis Q and q, factorization formulas include deformed numbers with sequence of powers of Q and q. As a specific case with $Q=\varphi$ and $q=\varphi'$ being the Golden and the Silver ratio, we get factorization formula for Fibonacci number F_N with arbitrary positive integer $N = p_1^{k_1}p_2^{k_2}...p_n^{k_n}$ to the set of integer numbers

$$F_N = \prod_{m_1=0}^{k_1-1}\prod_{m_2=0}^{k_2-1}...\prod_{m_n=0}^{k_n-1} F_{p_1}^{(p_1^{m_1})}\cdot F_{p_2}^{(p_1^{k_1}p_2^{m_2})}\cdot ...\cdot F_{p_n}^{(p_1^{k_1}p_2^{k_2}...p_{n-1}^{k_{n-1}}p_n^{m_n})}$$

This set of integers is represented by higher Fibonacci numbers [1] or Fibonacci divisors $F_{p_l}^{(k)}$ [2] of prime numbers p_l, with index k given by powers of p_l. Due to divisibility of Fibonacci number F_{nk} by F_k, so that $F_{nk}:F_k = F_n^{(k)}$, this factorization gives product of positive integer numbers.

Keywords Quantum calculus · Fibonacci numbers · Fibonacci divisors · Factorization

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ICMASE 2025

UNCERTAINTY RELATIONS AND ENTANGLEMENT FOR PQ-DEFORMED SUPERSYMMETRIC COHERENT STATES

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ABSTRACT

We propose supersymmetric extension of deformed quantum oscillator with two parameters quantum group structure. As particular cases, specified by values of P and Q parameters it includes symmetric and non-symmetric q-oscillators, Fibonacci and Fibonacci divisors hierarchy of Golden oscillators, Tamm-Dankov oscillator etc. By PQ-deformed supersymmetric annihilation operator, the set of corresponding supersymmetric coherent states is introduced. The states are characterized by the pair of PQ-quantum states from the Fock space or equivalently, by the set of infinite number of qubit states. Entanglement of fermions with PQ-deformed bosons is characterized by the concurrence as the linear entropy, taking form of the Gram determinant of inner products. As shown, for two types of the reference states, the concurrence depends on values of P or Q parameters, which are equal to one for the maximally entangled states. Entanglement of the super-coherent states and the uncertainty relations for the coordinate and momentum in these states are calculated. Non-classical nature of the entangled states is reflected in non-minimal character of the uncertainty relations.

***K*eywords** Coherent states · Supersymmetry · Quantum group · Entanglement

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ICMASE 2025

COMMON FIXED POINT THEOREM IN PARTIALLY ORDERED INTUITIONISTIC $\mathcal{L}$-FUZZY VECTOR METRIC SPACES

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ABSTRACT

Real-world applications of natural sciences frequently involve ambiguous or ill-defined solutions, giving rise to inherent fuzziness. To deal with this, Zadeh in [8] suggested a rule to handle fuzziness in design. Goguen in [4] later added to Zadeh's work by looking at more general ordered structures beyond just the unit interval. Identifying the supremum and infimum of a fuzzy set typically necessitates that the underlying partially ordered set (poset) constitutes at least a complete lattice adhering to the distributive property. Aliprantis discussed the concept of ordered vector spaces in his excellent book Positive Operators [1]. He further contributed to the concept of Riesz spaces, along with several associated structures and properties. The concept of vector metric spaces, where distances are measured in Riesz spaces, was initially presented in [2]. In [3], the parameter s was considered as a vector within the framework of L-fuzzy sets introduced by Goguen [4], alongside the fuzzy metric space formulated by Kramosil and Michálek [5]. To support this framework, the order structure was incorporated into the concept of left-hand continuity. Consequently, left and right order continuity notions were introduced to develop L-fuzzy vector metric spaces and their non-Archimedean variants. On the other hand, in [6] Banach contraction theorem in M-complete non-Archimedean fuzzy metric spaces was proved and the class of fuzzy contractive mappings was enlarged. In [7] the common fixed point theorem in $\mathcal{L}$-fuzzy metric spaces for arbitrary t-norms was proved. This work make a contribution to the theoretical development of fuzzy vector metric spaces and intuitionistic $\mathcal{L}$-fuzzy metric spaces constructed over order structures extending beyond the classical unit interval. It formalized the notions of left and right order convergence and continuity in an intuitionistic non-Archimedean $\mathcal{L}$-fuzzy vector metric space. In addition, it illustrates the proposed definitions and establishes several well-known results. Furthermore, it proves the common fixed point theorem in partially ordered intuitionistic $\mathcal{L}$-fuzzy vector metric spaces for commuting mappings. Finally, it addressed the necessity for further research in this area.

Keywords Intuitionistic $\mathcal{L}$-fuzzy vector metric · The common fixed point theorem · Commuting mappings · Riesz spaces

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ICMASE 2025

A GENERALIZATION OF R-SUPPLEMENTED MODULES

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ABSTRACT

In this work, every ring has an identity and every module over a ring R is a unitary left $R-$module. Let M be an $R-$module and $N \leq M$. If $L = M$ for every submodule L of M such that $M = N+L$, then N is called a small (or superfluous) submodule of M and denoted by $N \ll M$. Let M be an $R-$module and $U, V \leq M$. If $M = U + V$ and V is minimal with respect to this property, or equivalently, $M = U + V$ and $U \cap V \ll V$, then V is called a supplement of U in M. M is called a supplemented module if every submodule of M has a supplement in M. The intersection of all maximal submodules of an R-module M is called the radical of M and denoted by $RadM$. If M have no maximal submodules, then the radical of M is defined by $RadM = M$. Let M be an $R-$module and $U, V \leq M$. If $M = U + V$ and $U \cap V \ll M$, then V is called a weak supplement of U in M. M is said to be weakly supplemented if every submodule of M has a weak supplement in M. Let M be an $R-$module and $K \leq M$. If $K \ll RadM$, then K is callled an r-small submodule of M and denoted by $K \ll_r M$. Let M be an $R-$module and $U, V \leq M$. If $M = U + V$ and $U \cap V \ll_r V$, then V is called an r-supplement of U in M. If every submodule of M has an r-supplement in M, then M is called an r-supplemented module. Let M be an $R-$module and $U, V \leq M$. If $M = U + V$ and $U \cap V \ll_r M$, then V is called a weak r-supplement of U in M. If every submodule of M has a weak r-supplement in M, then M is called a weakly r-supplemented module. In this work, some properties of weakly r-supplemented modules are investigated. It is clear that every r-supplemented module is weakly r-supplemented. Because of this weakly r-supplemented modules are more general than r-supplemented modules.

Keywords Small Submodule · Radical · Supplemented Module · r-Supplemented Module

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GENERALIZED FOCAL CURVES OF HELICAL CURVES

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ABSTRACT

In the work [11] published in 2005, Uribe-Vargas gave the notions of focal curve and focal curvatures of a Frenet curve in $m+1$-dimensional Euclidean space $\mathbb{E}^{m+1}$, $m \geqq 2$. For any spherical curve, the focal curve degenerates at the center of the sphere that contains the curve. That is why we study generalized focal curves of spherical curves. We investigate the relations between the Frenet frames and the differential-geometric invariants of a helical curve in three-dimensional Euclidean space $\mathbb{E}^3$ and its closely related curve in four-dimensional Euclidean space $\mathbb{E}^4$. To obtain the parametric representation of the generalized focal curve of a helical curve, the focal curve of the constructed four-dimensional curve is used. Only helical curves, also known as curves with constant slope, will be examined in this work. The case of non-helical curves is studied in [4]. We construct a new curve in $\mathbb{E}^4$ (4D-curve) that is associative to a given space curve in $\mathbb{E}^3$. Then we find the focal curve of the corresponding 4D-curve. The orthogonal projection of the obtained curve in $\mathbb{E}^3$ is called a generalized focal curve of the initial curve. We provide a few helical spherical curve examples to demonstrate the results that were obtained.

Let $I \subseteq \mathbb{R}$ be a zero-containing interval, and let $\boldsymbol{\alpha} : I \longrightarrow \mathbb{E}^3$ be a helical Frenet curve of class C^4 with an arc-length parametrisation and a parametrical equation

$$\boldsymbol{\alpha}(s) = \big(x(s),\, y(s),\, a.s\big)^T, \quad s \in I,\, a = const,\, a \in (-1,0) \cup (0,1).$$

The unit-speed curve $\boldsymbol{\gamma}$ that is closely related to $\boldsymbol{\alpha}$ were studied in [9] and has a parametric equation

$$\boldsymbol{\gamma}(s) = \big(x(s),\, y(s),\, \cos(a.s),\, \sin(a.s)\big)^T, \quad s \in I.$$

The curve γ is a well-defined non-spherical Frenet curve with Frenet frames, a focal curve, and focal curvatures if α is a spherical space curve.

***K*eywords** Frenet curves · Helical curves · Focal curves · Spherical curves

Funding Research is partially supported by Konstantin Preslavsky University of Shumen under grant RD-08-53/22.01.2025.

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ON T-SUPPLEMENTED MODULES

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ABSTRACT

In this work, all rings have identities and all modules over a ring R are unitary left $R-$modules. Let M be an $R-$module and $N \leq M$. If $L = M$ for every submodule L of M such that $M = N + L$, then N is called a small (or superfluous) submodule of M and denoted by $N \ll M$. A submodule N of an $R-$module M is called an essential submodule and denoted by $N \trianglelefteq M$ in case $K \cap N \neq 0$ for every submodule $K \neq 0$, or equvalently, $N \cap L = 0$ for $L \leq M$ implies that $L = 0$. Let M be an $R-$module and $U, V \leq M$. If $M = U + V$ and V is minimal with respect to this property, or equivalently, $M = U + V$ and $U \cap V \ll V$, then V is called a supplement of U in M. M is said to be supplemented if every submodule of M has a supplement in M. If every essential submodule of M has a supplement in M, then M is called an essential supplemented (briefly, e-supplemented) module. Let M be an $R-$module. The radical of M is defined by the intersection of all maximal submodules of M and denoted by $RadM$. If M have no maximal submodules, then the radical of M is defined by $RadM = M$. Let M be an $R-$module. If every submodule of M which contains $RadM$ has a supplement in M, then M is called a strongly radical supplemented module. Let M be an $R-$module and $T \leq M$. M is called a T-supplemented module if every submodule of M which contains T has a supplement in M. In this work, some properties of T-supplemented modules are investigated. Let M be an $R-$module and $T \leq M$. If M is supplemented, then clearly we can see that M is T-supplemented. Because of this T-supplemented modules are more general than supplemented modules. Let M be an $R-$module and $T \leq M$. If M is T-supplemented and $T = RadM$, then M is strongly radical supplemented.

***K*eywords** Small Submodule · Radical · Supplemented Module · r-Supplemented Module

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MATHEMATICS DIAGNOSTIC IN ENGINEERING AND INSTRUCTIONAL DESIGN OF THE LOGICAL AND MATHEMATICAL SKILLS WORKSHOP

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ABSTRACT

In the context of engineering, first year students find it difficult to cope with the high mathematical content of their degree, a large number of students find mathematics modules difficult in the first year and, as a result, fail them. This hampers progression, retention and also the more demanding advanced mathematics courses required in subsequent years.

The results of the mathematics diagnostic test administered in 2024 to first-year engineering students at Universidad San Sebastián (Chile) revealed significant gaps in fundamental areas such as operations with real numbers, basic algebra, equation solving, and problem-solving. This situation prompted the implementation of an innovative proposal called *Taller de Aptitudes Lógicas y Matemáticas* (TALM), a course designed to strengthen students' foundational mathematical competencies and enhance academic progression through an integrated approach.

TALM combines a face-to-face component with an online module. The in-person component focuses on the development of mathematical skills through the use of contextualised exercises, game-based learning, and challenge-solving activities aimed at fostering active and collaborative learning that aligns with students' future professional practice. In parallel, the online component addresses metacognitive and self-regulation aspects of learning (such as time management, motivation, and study strategies), which are critical to sustaining academic effort during the transition to university life.

This study presents a description of the instructional design of the TALM. It highlights the course's innovative and adaptive approach, as well as its potential for replication in other contexts facing similar levelling challenges. The initiative forms part of a broader strategy to improve student retention, promote equity, and enhance the quality of teaching in engineering programmes.

***K*eywords** academic levelling · engineering · integrated mathematics education · active learning · games and challenges · contextualisation

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ADVANCED STATISTICAL TECHNIQUES FOR HEMATOLOGICAL DATA

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ABSTRACT

Hematologic diseases comprise a diverse set of disorders affecting the cellular components of the blood, including leukemias, lymphomas, myelomas and myelodysplastic syndromes, among others. These diseases significantly impact the physical, emotional, and social well-being of patients, requiring comprehensive care that combines biomedical treatment with emotional support and access to reliable information. This study presents an applied research project, developed through a Service-Learning experience, in which university students analyzed clinical and psychosocial data of hematological patients and their caregivers provided by the association AELCLÉS (Spanish Network of Entities Against Leukemia and Blood Diseases). This association is a non-profit group that was created from a group of entities related to blood diseases, in order to help restore the health of people affected by oncohematological diseases and support their families and caregivers. In order to make leukemia and other hematological diseases more visible, and to raise public awareness of the importance of blood, bone marrow and umbilical cord blood donation, a structured survey was passed to patients and caregivers to collect key information about their clinical situation, the emotional impact they experience, the support resources available to them and their unmet needs, with the ultimate goal of designing actions to improve their quality of life and strengthen the psychosocial support network.

The database, which includes 234 records and 60 variables related to sociodemographic, clinical, emotional and behavioral aspects, was cleaned and analyzed by students of different degrees under academic supervision. First-year statistics students were in charge of data cleaning and preparation. The statistical analyses were distributed according to academic profile: Engineering students applied descriptive techniques to characterize the sample; second-year Labor Relations students carried out inferential tests to identify significant differences and relationships between variables; finally, fourth-year Statistics students developed multivariate analyses, highlighting Multiple Correspondence Analysis (MCA) and logistic biplots, which allowed the identification of relevant associations between clinical and emotional variables. The analyses were performed with statistical software such as R, Python and SPSS.

The results revealed a greater representation of women in the sample, with an age distribution covering a wide range of ages. There were differences between sexes in the perception of immunological status and in the manifestation of emotional sequelae. Likewise, territorial variations in the prevalence of the different hematological diagnoses were observed. Multivariate analyses made it possible to identify key variables and explore the interrelationships in the data set, thus contributing to a better understanding for the possible personalization of therapeutic interventions and the strengthening of psychosocial support. This experience highlights the potential of Service-Learning to integrate

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rigorous technical training with the generation of applied knowledge of high social value, promoting scientific competencies and ethical commitment in students.

***K*eywords** Hematological diseases · Service-Learning · Statistical analysis · Multivariate analysis · Patient profile · Psychosocial care

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PREY REFUGE BEHAVIOR IN RESPONSE TO PREDATOR PRESENCE: IMPLICATIONS FOR A LESLIE-GOWER TYPE PREDATION MODEL

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ABSTRACT

In this work, we explore a continuous-time predator-prey model of the Leslie-Gower type, extended to include a biologically meaningful mechanism: the use of a physical refuge by a fraction of the prey population. Specifically, we assume that the proportion of prey individuals that access the refuge is directly proportional to the number of predators present in the environment. This assumption introduces a nonlinearity that significantly influences the dynamics of the system.

We analytically investigate the conditions under which equilibrium points exist and assess their local stability. Our analysis reveals that the origin, representing the extinction of both species, plays a central role in the dynamics. It generates a separatrix curve in the phase plane that divides the space of initial conditions into distinct behavioral regions. Trajectories that start above this separatrix tend to the origin, indicating that both populations face extinction in the long term. On the other hand, trajectories that originate below the separatrix may converge to a positive interior equilibrium, reflecting stable coexistence, or exhibit sustained oscillatory behavior in the form of a stable limit cycle.

This dual behavior highlights the importance of initial population sizes and the strength of the refuge effect in determining long-term outcomes. The presence of a refuge can help sustain prey populations under predation, but under certain circumstances, it may also contribute to the extinction of both species if the refuge is overutilized or ineffective due to high predator densities.

To complement the theoretical findings, we provide numerical simulations that illustrate the model's dynamics for different parameter values and initial conditions. These simulations confirm the analytical predictions and offer insight into how small changes in initial populations or refuge efficiency can lead to markedly different ecological scenarios. Our results underscore the ecological relevance of refuge mechanisms and contribute to a deeper understanding of predator-prey interactions.

***K*eywords** Predator-prey dynamics · Leslie-Gower model · Refuge · Equilibrium stability · Nonlinear system

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A LESLIE-GOWER TYPE PREDATION MODEL WITH NON-MONOTONIC FUNCTIONAL RESPONSE AND STRONG ALLEE EFFECT ON PREY

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ABSTRACT

In this work, a modified predator-prey model is analyzed, considering the Allee effect. To describe the dynamics of the model, we demonstrate the existence of a positively invariant region, the boundedness, and the permanence of the trajectories. We provide necessary and sufficient conditions for the existence and explicit form of up to two positive equilibria. One equilibrium is always a hyperbolic saddle, while the other can be an attractor, repeller, or weak focus. Additionally, we find two key scenarios: (i) a separatrix curve on the phase plane dividing the behavior of trajectories into qualitatively distinct regions, and (ii) a homoclinic curve generated by the stable and unstable manifolds of a saddle point in the interior of the first quadrant. These structures highlight the system's sensitivity to initial conditions, particularly near the separatrix. Bifurcations can occur in the system, including Hopf bifurcations, which further influence the model's dynamics. Finally, numerical simulations are presented to validate the analytical results.

***K*eywords** Predator-prey model · Leslie-Gower · Strong Allee efect · Nonmonotonic functional response · Equilibrium analysis

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OPTIMIZATION OF MAYER FUNCTIONAL IN OPTIMAL CONTROL PROBLEM WITH DISCRETE INCLUSIONS

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ABSTRACT

The optimization of Mayer functional in optimal control problems with discrete inclusions plays a crucial role in mathematical modeling and engineering applications. This study explores necessary and sufficient optimality conditions for discrete inclusions, incorporating Euler-Lagrange inclusions and transversality conditions. Using locally adjoint mapping techniques, we derive Euler–Lagrange and Hamiltonian-type conditions to establish a framework for solving discrete problems. The results contribute to the advancement of optimal control methodologies, offering new perspectives on constrained mathematical programming problems.

***K*eywords** Discrete inclusions · Optimality Conditions

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DYNAMICAL ANALYSIS AND OPTIMAL CONTROL OF A MATHEMATICAL MODEL FOR RED PALM WEEVIL INFESTATION

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ABSTRACT

In this study, we present a mathematical model to describe the population dy namics of the red palm weevil, a serious pest of palm trees. The most dangerous stage of this insect is the larval stage, which destroys the internal tissue of the palm. The mathematical model describes the different stages of the red palm weevil. The model integrates the main biological interactions and environmental factors that affect the insect's life cycle. We perform a comprehensive dynamic analysis of the model, identifying equilibrium points, evaluating their stability, and studying bi furcation. Overall stability is analyzed using the appropriate Lyapunov function and employing the geometric approach to investigate stability using the Lozinski measure. A sensitivity analysis is conducted to determine the influence of various factors on the system's behavior. Furthermore, we apply the Pontryagin princi ple of extreme values to formulate an optimal control problem aimed at reducing infestations through targeted interventions, such as pesticides and biological con trol. Numerical simulations are performed using the forward and backward sweep method. The results indicate the effectiveness of different control strategies and their impact on sustainable pest management in palm farms.

***K*eywords** Optimal Control · Red Palm Weevil · Lozinski Measure Mathematical Models

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SOME SPECIAL MATRICES WITH FIBONOMIAL COEFFICIENTS

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ABSTRACT

In this talk, we will discuss some matrices whose elements are different types of Fibonomial coefficients and various properties of these matrices.

Keywords Special Matrices · Fibonacci numbers · Binomial expansion

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ICMASE 2025

ON SEMIGROUPS OF TRANSFORMATIONS WHOSE RESTRICTIONS ARE ELEMENTS OF A GIVEN SEMIGROUP

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ABSTRACT

Let $n \in \mathbb{Z}^+$ and let $X_n = \{1, \ldots, n\}$. As it is well known that P_n, T_n, I_n and S_n denote the partial transformation semigroup, (full) transformation semigroup, symmetric inverse semigroup (the semigroup of all partial one to one maps) and symmetric group (group of all permutations) on the set X_n, respectively. Now, let Y be a non-empty subset of the set X_n with cardinality m $(1 \leq m \leq n-1)$. Without loss of generality, we can consider the set X_m, rather than Y and define the semigroups

$$\begin{aligned}
IS_{(n,m)} &= \{\, \alpha \in I_n : \alpha_{|_{X_m}} \in S_m \,\}, & II_{(n,m)} &= \{\, \alpha \in I_n : \alpha_{|_{X_m}} \in I_m \,\},\\
TT_{(n,m)} &= \{\, \alpha \in T_n : \alpha_{|_{X_m}} \in T_m \,\}, & PS_{(n,m)} &= \{\, \alpha \in P_n : \alpha_{|_{X_m}} \in S_m \,\},\\
PT_{(n,m)} &= \{\, \alpha \in P_n : \alpha_{|_{X_m}} \in T_m \,\}, & PI_{(n,m)} &= \{\, \alpha \in P_n : \alpha_{|_{X_m}} \in I_m \,\},\\
PP_{(n,m)} &= \{\, \alpha \in P_n : \alpha_{|_{X_m}} \in P_m \,\}
\end{aligned}$$

for $1 \leq m \leq n-1$. In this study, we introduce and examine these new semigroups.

Keywords Partial (Full) transformation semigroup · Symmetric inverse semigroup · Symmetric group

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This is a joint work with Rukiye Sönmez and Hayrullah Ayık.

ON SEMIGROUPS OF TRANSFORMATIONS WITH IDEMPOTENT COMPLEMENT WHOSE RESTRICTIONS BELONG TO A GIVEN SEMIGROUP

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ABSTRACT

Let $X_n = \{1, \ldots, n\}$ for any $n \in \mathbb{Z}^+$, and let P_n, T_n, I_n and S_n denote the partial transformation semigroup, (full) transformation semigroup, symmetric inverse semigroup and symmetric group on the set X_n, respectively. In this study, we introduce and examine some new semigroups defined as

$$\begin{aligned}
IS^f_{(n,m)} &= \{\alpha \in I_n : \alpha_{|_{X_m}} \in S_m;\ \mathrm{dom}(\alpha) \setminus X_m \subseteq \mathrm{fix}(\alpha)\},\\
II^f_{(n,m)} &= \{\alpha \in I_n : \alpha_{|_{X_m}} \in I_m;\ \mathrm{dom}(\alpha) \setminus X_m \subseteq \mathrm{fix}(\alpha)\},\\
PS^f_{(n,m)} &= \{\alpha \in P_n : \alpha_{|_{X_m}} \in S_m;\ \mathrm{dom}(\alpha) \setminus X_m \subseteq \mathrm{fix}(\alpha)\},\\
PT^f_{(n,m)} &= \{\alpha \in P_n : \alpha_{|_{X_m}} \in T_m;\ \mathrm{dom}(\alpha) \setminus X_m \subseteq \mathrm{fix}(\alpha)\},\\
PI^f_{(n,m)} &= \{\alpha \in P_n : \alpha_{|_{X_m}} \in I_m;\ \mathrm{dom}(\alpha) \setminus X_m \subseteq \mathrm{fix}(\alpha)\},\\
PP^f_{(n,m)} &= \{\alpha \in P_n : \alpha_{|_{X_m}} \in P_m;\ \mathrm{dom}(\alpha) \setminus X_m \subseteq \mathrm{fix}(\alpha)\}
\end{aligned}$$

for $1 \leq m \leq n-1$ where $\mathrm{dom}(\alpha) = \{x \in X_n : x\alpha = y \text{ for any } y \in X_n\}$ and $\mathrm{fix}(\alpha) = \{x \in \mathrm{dom}(\alpha) : x\alpha = x\}$.

***K*eywords** Idempotent element · Partial (Full) transformation semigroup · Symmetric inverse semigroup · Symmetric group

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DESCRIPTIVE PROXIMAL RELATOR SPACES: ADVANCES IN THE ALGEBRA OF PRIME RINGS AND IDEALS

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ABSTRACT

Given a non-empty set X and its subsets A and B, we say A is near to B (denoted by $A\delta B$), which implies that either A and B have one or more common points, or they have points close enough to each other [7]. A non-empty set X, equipped with δ is called a proximity space, provided it satisfies certain conditions. The nearness between two sets does not necessarily mean that there are common elements between the two sets *(Spatially proximity)*. Instead, nearness can be defined descriptively as *(Descriptive proximity)* [1, 6] based on probe functions [4]. A family of proximity relations R_δ together with a non-empty set X, defines a relator space [5]. Many algebraic structures have been proposed for relator spaces, such as approximately groups [2], approximately rings, approximately sub-rings, approximately ideals [2, 3]. These structures play an important role in algebraic theory. These observations have led lead to advances in descriptive relator space theory, namely, approximately prime rings (APRs) and approximately prime ideals (APIs). In this study, we present APRs and APIs in descriptive relator spaces and together with some of their properties. Moreover, we give a main theorem related to the approximately integral domain.

***K*eywords** relator spaces · approximately rings · approximately prime rings

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ICMASE 2025

NEUTROSOPHIC 2-ABSORBING PRIMARY IDEALS IN LATTICE STRUCTURES

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ABSTRACT

This study introduces the concepts of neutrosophic prime ideal, prime neutrosophic ideal, neutrosophic 2-absorbing, 2-absorbing neutrosophic ideal, neutrosophic primary ideal, and primary neutrosophic ideal on a lattice. Then, neutrosophic 2-absorbing primary ideals of a lattice are characterized. The transitions among these concepts and their relationships with the ideals of a lattice are discussed. Finally, these concepts are examined on a lattice formed by the Cartesian product of lattices.

***K*eywords** Lattice, neutrosophic, prime, 2-absorbing, primary

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ON FIXED POINT RESULTS FOR HYBRID-INTERPOLATIVE REICH-ISTRĂŢESCU-TYPE CONTRACTIONS WITHIN THE FRAMEWORK OF SOFT METRIC SPACES

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ABSTRACT

In recent decades, the study of uncertainty has garnered substantial attention across various disciplines, leading to the development of several mathematical tools. Among these, soft set theory, introduced by Molodtsov in 1999 [1], has emerged as a significant framework for addressing problems characterized by imprecise, ambiguous, or incomplete information. Its flexible and comprehensive structure has led to a growing body of literature exploring its theoretical and applied dimensions. Concurrently, the generalization of classical metric space theory has been a focal point of research. Scholars have proposed numerous generalized metric structures by either modifying the axioms of metric spaces or extending the nature of the set over which the metric is defined. One such innovative structure is the soft metric space, which integrates the principles of soft set theory into the framework of metric spaces, offering a novel and effective approach for analyzing problems involving uncertainty.

A particularly vibrant area within the theory of metric and generalized metric spaces is fixed point theory, which investigates the conditions under which mappings admit fixed points. The existence and uniqueness of such points have profound implications in mathematics and related fields. In this context, various contractive conditions have been formulated to generalize the classical Banach contraction principle. Among recent advancements, Karapınar et al. (2022) [2] introduced a new class of contractive mappings known as hybrid-interpolative Reich-Istrăţescu-type contractions. These mappings are characterized by their dependence on multiple parameters, allowing for a more flexible and encompassing framework. Their formulation provides a unifying structure that generalizes several well-known contractive mappings, and hence, has potential applications in diverse generalized metric settings, including soft metric spaces.

This paper aims to extend the applicability of hybrid-interpolative Reich-Istrăţescu-type contractions by formulating and analyzing them within the framework of soft metric spaces. To this end, we first provide a rigorous definition of this class of contractions in the context of soft metric spaces, incorporating the notion of soft points and soft mappings. Subsequently, we establish a series of fixed point theorems under these generalized contractive conditions. The existence and uniqueness results are derived using novel analytical techniques tailored to the soft metric framework. Furthermore, illustrative examples are presented to demonstrate the applicability and effectiveness of the theoretical results obtained. These examples validate the assumptions and highlight the practical relevance of the proposed theorems.

The findings of this study contribute significantly to the growing literature on fixed point theory in generalized metric spaces. By extending the concept of hybrid-interpolative contractions to soft

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metric spaces, we not only enrich the theoretical foundation of soft analysis but also open new avenues for future research in the area of non-classical metric structures. The results presented herein are expected to have implications in fields where uncertainty modeling and decision-making under vagueness are essential, including optimization, control theory, and computational mathematics.

***K*eywords** Hybrid-interpolative Reich-Istra¸tescu-type contractions, soft metric spaces, soft points, soft mappings

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ICMASE 2025

NOVEL FIXED POINT RESULTS VIA HYBRID-INTERPOLATIVE REICH-ISTRĂŢESCU-TYPE CONTRACTIONS IN PARAMETRIC S-METRIC SPACES

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ABSTRACT

Fixed point theory has long been recognized as a cornerstone of modern mathematical analysis, particularly in the context of nonlinear analysis, topology, and applications to differential equations, dynamical systems, and optimization. Among its classical results, Brouwer's Fixed Point Theorem [1] and Banach's Contraction Principle [2] have laid a strong foundational framework. Banach's theorem, in particular, ensures the existence and uniqueness of fixed points for contraction mappings on complete metric spaces, providing the theoretical underpinning for many iterative and convergence-based solution methods in applied mathematics.

As the limitations of classical metric spaces became evident in modeling complex and uncertain systems, various generalizations of the metric structure have been introduced. These include but are not limited to G-metric spaces, b-metric spaces, and S-metric spaces. The concept of an S-metric space, proposed by Sedghi et al. [3], is a notable extension of the classical metric space. In an S-metric space, the distance function is defined on three variables rather than two, offering greater analytical flexibility and an alternative approach for studying convergence and fixed point behavior. The advent of parametric S-metric spaces has engendered a novel analytical environment, thereby enhancing our capacity to address fixed point problems in uncertain or imprecise contexts.

In parallel, the generalization of contraction conditions has also received significant attention in recent literature. Various generalizations such as Kannan [4], Chatterjea [5], Reich [6], and Istrăţescu [7] contractions have been proposed to relax the strict assumptions of Banach's original formulation. Building upon this line of work, Karapınar et al. [8] introduced the hybrid-interpolative Reich-Istrăţescu type contraction, a flexible, parameter-dependent contractive mapping that unifies and generalizes multiple existing contraction types.

This study aims to contribute to the expanding field of metric fixed point theory by investigating fixed point results within the framework of parametric S-metric spaces using the hybrid-interpolative Reich-Istrăţescu contraction. To this end, we adapt and redefine the contraction condition introduced by Karapınar et al. [8] to suit the structure of parametric S-metric spaces. We then establish several new fixed point theorems under this generalized setting. The validity and applicability of the theoretical findings are demonstrated through illustrative examples.

By unifying the generalized metric framework of S-metric spaces and parametric structures and the richness of hybrid contraction mappings, this research significantly extends the current scope of fixed point theory. The results not only generalize existing theorems but also pave the way for further investigations into applications where uncertainty and higher-order metric structures play a central role.

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***K*eywords** Hybrid-interpolative Reich-Istra¸tescu-type contractions, parametric S-metric spaces, S-metric spaces

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ICMASE 2025

NULL HYBRID (1,3)-BERTAND CURVES

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ABSTRACT

The concept of hybrid numbers, introduced by Özdemir [13], generalizes complex, hyperbolic, and dual numbers. A hybrid number is expressed as $H = h_0+h_1\mathbf{i}+h_2\boldsymbol{\epsilon}+h_3\mathbf{h}$ where $h_0, h_1, h_2, h_3 \in \mathbb{R}$ and $\mathbf{i}\times\mathbf{i} = -1, \boldsymbol{\epsilon}\times\boldsymbol{\epsilon} = 0, \mathbf{h}\times\mathbf{h} = 1, \mathbf{i}\times\mathbf{h} = -\mathbf{h}\times\mathbf{i} = \boldsymbol{\epsilon}+\mathbf{i}$ are satisfied. The set of hybrid numbers is identified with the set of four-dimensional Minkowski space of index 2, $\mathbb{R}^4_2$, analogous to how the four-dimensional Euclidean space, $\mathbb{E}^4$, is identified with quaternions. Bharathi and Nagaraj [6] used quaternions to study the geometry of the curves in $\mathbb{E}^4$, and Akbıyık [1] extended this approach by using hybrid numbers to investigate non-null curves in $\mathbb{R}^4_2$.

Null curves are, on the other hand, of great importance in space-time geometry and physics. Consequently, extensive research has been done on null curves in Minkowski spaces of dimension n and index q. A comprehensive study on null curves in the (m+2)- dimensional manifolds of index q is provided by Duggal and Jin [8]. Alo [5] studied null hybrid curves and gave some properties of null hybrid Bertrand curves. They constructed a null Frenet frame for null hybrid curves using the null frame of the associated null spatial hybrid curve. They derived the null Frenet formulas and established relations between the curvatures of these two curves. Furthermore, they examined special null hybrid curves, null hybrid Bertrand curves, and showed that any such curve is a null spatial hybrid Bertrand curve- consistent with the fact that Bertrand curves in the space of dimension $n \geq 4$ are degenerate curves. This property of Bertrand curves in $\mathbb{E}^n$ $(n \geq 4)$ was initially proved by Matsuda and Yorozu [12], so they presented a new type of Bertrand curves in a space of dimension $n \geq 4$, called (1,3)-Bertrand curves. A curve $\hat{\Gamma}$ is said to be a (1,3)-Bertand mate of Γ if the normal plane spanned by the principal normal and the binormal of $\hat{\Gamma}$ coincides with the normal plan spanned by the principal normal and the binormal of Γ.

In this presentation, we investigate null hybrid (1,3)-Bertrand curves. We give conditions for a curve $\hat{\Gamma}$ to be a null hybrid (1,3)-Bertrand mate of the curve Γ. Furthermore, if γ is an associated null spatial hybrid curve of Γ, and $\hat{\gamma}$ a null Bertrand mate of γ, we examine the conditions under which the curve $\hat{\Gamma}$, whose associated curve is $\hat{\gamma}$, is a null hybrid (1,3)-Bertrand mate of Γ.

***K*eywords** Hybrid Numbers · Null Hybrid Curves · (1,3)-Bertrand Curves

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ICMASE 2025

FIBERED CATEGORIES: 2-GENERALIZED CROSSED MODULES

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ABSTRACT

Algebraic models of homotopy types are essential tools in homotopical algebra and higher category theory. In this context, 2-crossed modules were introduced by Conduché [3] as a categorical and algebraic model for connected homotopy 3-types, extending the classical notion of crossed modules that model homotopy 2-types.

The structure of a 2-crossed module consists of group homomorphisms equipped with compatible actions and Peiffer liftings, where the actions are typically given by conjugation. To extend the reach of this framework, the concept of a 2-generalized crossed module was defined in [7], allowing arbitrary group actions rather than restricting to conjugation. This generalization provides a more flexible algebraic setting for modeling higher-dimensional homotopical structures.

Let $\mathcal{F}$ and $\mathcal{Z}$ be two categories, and let $\Omega\colon \mathcal{F} \to \mathcal{Z}$ be a functor. We say that Ω is a category fibred over $\mathcal{Z}$ via Ω if and only if every morphism $\alpha\colon Z' \to Z$ in $\mathcal{Z}$ and every object $F \in \mathcal{F}$ such that $\Omega(F) = Z$, there exists a cartesian morphism $\beta\colon F' \to F$ in $\mathcal{F}$ such that $\Omega(\beta) = \alpha$. This condition allows us to "pullback" objects along morphisms in the base category, preserving structure.

The notion of fibered category was introduced by Grothendieck in [5] to formalize geometric ideas in descent theory by organizing objects and morphisms over a base category. This framework was later developed in detail by Bénabou [1], explaining its logical foundations and emphasized its role in category theory over a base with pullbacks. Fibered categories also offer a natural setting to study algebraic models of homotopical structures such as crossed modules, which play a key role in modeling homotopy 2-types [2]. Together, these concepts form essential tools in higher category theory and modern algebraic topology [1, 8].

The fibration of the category of 2-crossed modules over groups was investigated in [4], while fibrations and cofibrations of generalized crossed modules were introduced in [6].

In this work, it is shown that the category of 2-generalized crossed modules is a fibered category.

***K*eywords** Fibered category · Generalized crossed module · Pullback

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REVERSIBILITY OF RINGS WITH RESPECT TO THE ZHOU RADICAL

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ABSTRACT

In my talk, I will present the results from the article *"Reversibility of Rings with Respect to the Zhou Radical,"* coauthored with T. Calci. Let R be a ring with identity, and let $\delta(R)$ denote the Zhou radical of R. We define a ring R to be *δ-reversible* if, for all $a, b \in R$, the condition $ab = 0$ implies $ba \in \delta(R)$. In this presentation, I will discuss several properties of δ-reversible rings and examine various ring extensions that preserve δ-reversibility.

***K*eywords** Zhou radical · Reversible ring · δ-reversible ring · Ring extension

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SOCIAL BEHAVIORS IN BOTH INTERACTING SPECIES AND THE ALLEE EFFECT. THEIR IMPACT ON A PREDATION MODEL.

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ABSTRACT

In a predation interaction between two species, considering different ecological phenomena can produce significant changes in the dynamics of the models that describe it. Among these phenomena, the use of physical refuges by prey as a defense strategy against the constant threat of predation stands out. This behavior, directly influenced by the perceived level of risk, alters the probability of encounter between predator and prey.

Another relevant phenomenon is intraspecific competition among predators, driven by different causes, such as space limitation, depletion of food, etc. As predator density increases, the fight for access to prey intensifies. The Allee effect on prey postulates that the per capita growth rate of prey is reduced for low population sizes. This can have significant consequences for population survival and permanence.

Although there are many other ecological phenomena, the study and analysis of a mathematical model that incorporates the three phenomena mentioned above is proposed, adding as an assumption that the fraction of the prey population that uses the refuge is directly proportional to the number of predators present in the environment.

The system of differential equations proposed will be proved as well-posed and feasible as a population model. This will involve demonstrating the boundedness and positivity of the solutions, and analyzing the repulsive nature of the point $(\infty, 0)$, ensuring that the populations do not evolve towards physically impossible values and that the system avoids non-biological behaviors.

In addition, the existence and stability of equilibrium points of the system will be studied, representing scenarios where the predator population dies out and the prey population reaches a steady status. To complement this analysis, numerical simulations showing some possible dynamic scenarios will be used to visualize the global behavior of the system solutions and how population densities vary over time, under different combinations of parameters and initial conditions.

Keywords Predator-prey model · Intra-specific competition among predators · Allee effect · Refuge · Stability

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ICMASE 2025

A MALTHUSIAN PREDATOR-PREY MODEL WITH SATURATED FUNCTIONAL RESPONSE

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ABSTRACT

Mathematical models are widely used in ecology. Particularly in population ecology, predator-prey models have a long history since their proposal in the 1920's by mathematician Vitto Volterra, and there are numerous studies on them with in-depth analysis of their properties. The Gause-type of predator-prey models includes compartmentalized models in which biomass leaving the prey compartment enters the predator compartment with some eventual conversion efficiency. Several of these models consider ecological factors such as self-interference in prey or saturation of prey consumption by predators. One of them is the well-known Rosenzweig-MacArthur model, which incorporates the well-known logistic equation in the prey growth rate to include self-interference ecological factor in this compartment and a functional response to represent predator saturation factor. However, a type of predator-prey model that has been little studied in the literature is one that assumes unrestricted growth in prey. A Gause-type predation model with Malthusian prey growth and saturation in predators described by the Holling Type II functional response in hyperbolic form is proposed and studied in this work. The main results obtained are presented: the existence of a single positive equilibrium point (inside the first quadrant) that is unstable for all parameter values and the non-existence of limit cycles in the system. These results suggest that prey self-interference significantly influences the population dynamics predicted by the model, both in terms of the stability of equilibrium points and the existence of fluctuations in population size for both species. This assertion is based on a comparative study of the Rosenzweig-MacArthur model and the one presented in this paper.

***K*eywords** Predator-prey model · malthusian growth · limit cycle · separatrix curve · stability

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CLUSTERING OCCUPATIONAL SAFETY PROFESSIONALS' PERCEPTIONS ON EMERGING TECHNOLOGIES: A DATA-DRIVEN SEGMENTATION APPROACH

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ABSTRACT

The integration of emerging technologies—such as Artificial Intelligence (AI), Internet of Things (IoT), and wearable devices into Occupational Health and Safety (OHS) systems is transforming workplace risk management. However, adoption varies considerably depending on practitioners' perceptions and organizational contexts. This study applies unsupervised machine learning techniques to segment OHS professionals based on their perceptions of benefits and challenges associated with these technologies. Using a dataset from a national survey of Portuguese OHS technicians, we employ dimensionality reduction (PCA and t-SNE) followed by clustering algorithms (k-means and DBSCAN) to discover latent clusters among respondents. The analysis reveals different groups, including "technology enthusiasts", "cautious adopters" and "resistant sceptics", each with specific concerns such as data privacy, training needs, or ethical implications. These findings suggest that a one-size-fits-all strategy is insufficient for implementing safety technologies and highlight the value of perception-based segmentation for targeted interventions. Our work provides a methodological contribution to digital transformation efforts in OHS and practical guidance for policymakers and security managers who aim to tailor adoption strategies to specific professional profiles.

***K*eywords** occupational health and safety · clustering · unsupervised learning · perceptions · emerging technologies · t-SNE · k-means · segmentation · Portugal

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This work was supported by national funds through FCT—Fundação para a Ciência e Tecnologia through project UIDB/04728/2020.

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A SHORT NOTE ON r-SUBMODULES AND sr-SUBMODULES

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ABSTRACT

Let M be a module over a commutative ring R. Recall from [3] that a proper submodule N of M is said to be an r-submodule (special r-submodule, or briefly, sr-submodule) if whenever $am \in N$ with $ann_M(a) = 0_M$ ($ann(m) = 0$) for some $a \in R$ and $m \in M$, then $m \in N$ ($a \in (N : M)$). Recently, the notion of r-submodules and sr-submodules has attracted the attention of many researchers. For instance, in [1] and [2], Anebri et al. studied ascending and descending chain conditions on r-submodules. Afterwards, in [4] Mahdou et al. characterized modules satisfying Property (A) and Property (T) in terms of r-submodules and sr-submodules. In this study, we introduce a new generalization of r-submodules and use it to characterize some important classes of modules.

***K*eywords** r-submodule · sr-submodule · r-ideal

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A FUZZY CIMAS–FUZZY VIKOR METHODOLOGY FOR PRIORITIZING LOGISTICS WAREHOUSE MANAGEMENT PROBLEMS

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ABSTRACT

Warehouse management in logistics is critical for providing accurate and fast service, ensuring customer satisfaction, and sustaining the supply chain. However, various problems can arise in warehouse management, leading to significant losses for logistics firms. To minimize these losses under the constraints of limited resources, it is essential to first determine which of these problems should be prioritized for action. Such a strategic decision should be made by utilizing analytical approaches and evaluating the problems based on predetermined criteria. In this study, an integrated fuzzy multi-criteria decision-making (MCDM) model based on the Fuzzy CIMAS and Fuzzy VIKOR methods is used to prioritize the cost and loss criteria to be evaluated and to rank the common problems encountered in warehouse management in logistics. The data required for the study is obtained from an expert team consisting of individuals responsible for logistics and warehouse management. After prioritizing the problems, suggestions were presented to address those of highest importance. This study is the first in the logistics literature to address the decision-making problem and the proposed model under consideration.

Acknowledgements This study is supported by TUBITAK 2209-A - Research Project Support Programme for Undergraduate Students and Eskisehir Technical University, Scientific Research Projects Committee (25LOP024).

***K*eywords** MCDM · Fuzzy set · Logistics · Warehouse management

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POLYNOMIAL CODES OVER FINITE FIELDS

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ABSTRACT

The most general class of error detection and correction codes is the class of linear codes. A special class of linear codes are polynomial codes, where messages are encoded using polynomials with coefficients in a finite field. For correcting single errors, one big advantage of polynomial codes is that specifying only the first row of the encoder/ generator matrix G is sufficient to describe the entire code. These codes are widely used in digital communication and storage systems.

***K*eywords** Finite fields · polynomial codes · BCH codes

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ICMASE 2025

INTUITIVE APPROXIMATIONS FOR A RESIDUAL WAITING TIME PROCESS

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ABSTRACT

Let $W(t) \equiv T_{N(t)} - t$ denote the residual waiting time at time t, where $T_{N(t)} = \sum_{i=1}^{N(t)} X_i$ is the last renewal epoch before or at time t, and $\{X_n\}_{n\geq 1}$ is a sequence of idependent and identically distributed positive random variables generating a renewal process $N(t)$. The residual waiting time process occupies a central position in the analysis of stochastic systems, serving as a fundamental component in reliability theory, queueing models, and inventory dynamics. Its ability to quantify the time remaining until the next event renders it indispensable in modeling delays and forecasting performance in real-time systems. Accurate computation of its expected value is particularly critical in applications such as service operations, transportation planning, and communication networks, where delays translate directly into costs or inefficiencies.

Although its importance is well-recognized, the analytical study of $W(t)$ remains challenging.

In this work, we derive intuitive approximation formulas for the expected value of the residual waiting time process under two specific distributional settings:
(i) When the interarrival times X_n follow a regularly varying distribution with index $\alpha > 2$,
(ii) When X_n belongs to the $\Gamma(g)$ distribution family.
Regularly varying distributions play a central role in extreme value theory due to their heavy-tailed nature and their connection to the Fréchet domain of attraction. However, the regularly varying case does not include examples such as the exponential distribution or the gamma distribution. In extreme value theory, the class of $\Gamma(g)$ appears in a natural way to cover such light- or intermediate-tailed cases, including generalized extreme value and logistic distributions. To achieve these results, we utilize intuitive approximation techniques for the renewal function developed by Mitov and Omey. To the best of our knowledge, the literature does not seem to provide intuitive approximations for the residual waiting time in these settings.

***K*eywords** Residual waiting time process · Intuitive approximation · Renewal function · Regularly varying distributions · $\Gamma(g)$ class of distributions.

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VI International Conference on Mathematics and its Applications in Science and Engineering
(ICMASE 2025)

Emergence of Multirhythmicity in Cortical Networks with Two Types of Inhibition

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Abstract

We study a network model composed of three interacting neuronal populations: pyramidal (Pyr) cells, parvalbumin-positive (PV) interneurons, and somatostatin-positive (SST) interneurons. Starting from a network of globally coupled quadratic integrate-and-fire (QIF) neurons with heterogeneous inputs, we reduce the full spiking network to a low-dimensional mean-field model consisting of 9 ordinary differential equations (ODEs) — three for each population.

This reduced system captures the essential dynamics of the network, allowing for tractable bifurcation and phase space analysis. We demonstrate the emergence of multistability, oscillatory switching, and coexisting rhythms (mixed beta states) across the populations. In particular, we find that the strength and directionality of inter-population interactions critically depend on SST-IN-mediated inhibition, which modulates transitions between distinct beta-band oscillatory states.

Our results reveal how multiple subtypes of inhibitory neurons coordinate to generate and regulate complex beta dynamics, with potential implications for understanding neural mechanisms underlying motor control, cognitive function, and beta-band abnormalities observed in disorders such as Parkinson's disease.

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LINEAR ALGEBRA AND COMMUNICATION AND MULTIMEDIA STUDENTS

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ABSTRACT

Linear Algebra (LA) is a mathematical subject in almost all university programs. However, it has always proved difficult for students to learn. The main criticisms pointed out by students of the LA course concern the excessive use of formalism, the number of new definitions and results involved in learning a concept, and the lack of connection with mathematical knowledge already acquired. Also, some studies refer to emotional factors - attitudes and anxiety. In this context, to identify errors and difficulties and understand the students' reasoning when solving questions considered preparatory for learning LA, a questionnaire was given to 54 1st-year students who attended the LA course of Communication and Multimedia (CM) at a university in Portugal. This work was based on the framework of ontosemiotics by Godino and colleagues (OSA), who defined algebraization levels as the algebraic manipulations needed to develop algebraic thinking. In addition, we also considered elements of algebraic thinking: generalized arithmetic (GA), functional thinking (FT), modelling language (ML), abstract algebra (AA), and algebraic proof (AP). The questionnaire was given to establish the errors and difficulties in algebraic thinking through its elements. This qualitative, interpretative, and descriptive research aimed to identify which algebraic thinking elements the students had achieved. However, we found the presence of features of algebraic thinking in these CM students. The first question, referring to FT, had the highest percentage of correct answers (52 percent). In the following, the rate of correct answers decreased, and, above all, the absence of the requested justifications for the answers was noticeable. In the last two questions, referring to AP, less than two percent of the students did the appropriate algebraic manipulations but with incorrect justifications or arguments. Thus, students of this subject in higher education do not have the right level of algebraic thinking for learning LA.

***K*eywords** Algebraic thinking · Algebraization levels · Questionnaire

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THIN FILM FLOW DOWN A VERTICAL SUBSTRATE IN THE PRESENCE OF INTERFACE

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ABSTRACT

The hydrodynamics of a multi-layer flow have attracted the attention of many researchers because of the wealth and variety of waves which develop on multiphase interfaces. Besides experimental studies, the modelling of the problem has been widely studied in the literature but modelling of direct numerical simulations have been limited for moving faster wave families on various layer thicknesses for stationary film thickness. The bifurcation analyse of multi-layer flows with free surface is investigated by using approximate wave model. Calculations have been carried out to generate the bifurcation scheme which shows the families of various wave velocities. Examples of nonlinear wave shapes are illustrated at real-life values.

***K*eywords** fluid dynamics · multi-layer flows · thin films · waves · bifurcation

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MEASURING HAPPINESS IN HIGHER EDUCATION: ENGINEERING STUDENTS' INSIGHTS BASED ON AN ADAPTATION OF THE OXFORD HAPPINESS QUESTIONNAIRE

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ABSTRACT

Awareness of student well-being is even more relevant to higher education with the initiatives that universities are taking toward cultivating inclusive, empathetic, and caring learning communities [1, 2, 3, 4]. This study examined engineering students' experience of happiness at the Instituto Superior de Engenharia de Coimbra using the Portuguese translation of the Oxford Happiness Questionnaire [5] and a brief demographic and academic questionnaire.

186 students across educational levels and fields of study volunteered for the study. The sample characteristics were varied in the demographics: 86% of the participants indicated they were Portuguese and 44% indicated they resided in urban areas. In their personal lives, 52.2% indicated that they had not had a romantic relationship while in school. The outcomes indicated large differences in the perceived happiness based on the year of study, course being pursued, and students' level of academic satisfaction.

Item analysis revealed that while most students responded positively to questions on frequent laughter, regular joy, and sense of positive influence on others, others experienced low self-esteem, confusion concerning personal identity, and indecision. Many of the students provided equivocal responses to queries on optimism regarding the future, restfulness, and a sense of purpose in life, indicative of emotional challenges not necessarily evident from their academic performance.

These findings provide evidence for the inclusion of mental well-being assessment in higher education practice and institutional tools that assess and support students' social and emotional adjustment. Inducing a general learning approach that coordinates emotional well-being with academic achievement is the key to producing resilient, motivated, and satisfied students.

Keywords Student happiness · Well-being · Engineering education · Oxford Happiness Questionnaire · Academic satisfaction

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"PASSPORT TO LINEAR ALGEBRA: VECTORS AND MATRICES IN THE BAG!"- A GAMIFIED LEARNING EXPERIENCE FOR ENGINEERING STUDENTS

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ABSTRACT

To situate abstract mathematical concepts so that they are more relevant and engaging for engineering students, gamification has proven to be an innovative and successful strategy [1, 2, 3]. The proposed paper presents the experience "Passport to Linear Algebra: Vectors and Matrices in the Suitcase!", a gamified learning environment for first-year Biomedical and Electromechanical Engineering students at the Polytechnic Institute of Coimbra. The activity consists of creating a virtual team trip around the world, where students create and solve Linear Algebra problems. Each team receives a mission on a boarding pass and a luggage tag that identify the departure city and the arrival city. From here, students create a journey narrative that includes matrix operations, determinants, linear systems, eigenvalues and eigenvectors, and linear transformations while developing clues for the other teams. The tips, which aim to guide the team to a destination, must combine program content with imagination and contextualization.
49 students were involved in a full week of classes (4 hours). The teams collaborated to complete their mission, and the assessment was conducted through the clarity, creativity and interaction of the group in developing the problems presented in the "trip log" spreadsheet, in the formal correction of the solved problems, in the observation of the teacher in the classroom and in the pre/post questionnaires measuring the knowledge acquired and the student's attitude.
The results indicated high levels of conceptual and motivational knowledge. Students became more involved and gained a greater ability to connect theory with practice. The experience also promoted transversal skills, such as teamwork, communication and problem solving [3, 4]. This gamification, interdisciplinary learning experience demonstrates the power of active learning techniques in engineering education to make mathematics more interactive, relevant, and engaging.

Keywords Gamification · Linear Algebra · Engineering Education · Active Learning · Interdisciplinary Teaching

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NECESSARY AND SUFFICIENT CONDITION FOR INFINITE TIME BLOW UP OF GLOBAL SOLUTIONS TO WAVE EQUATION WITH LOGARITHMIC NONLINEARITY

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ABSTRACT

This work investigates global solutions to the initial boundary value problem for the wave equation with logarithmic nonlinearity in a bounded domain. A novel necessary and sufficient condition for infinite time blow up of global weak solutions is proved in the case of arbitrary positive initial energy. As a consequence, we derive a new sufficient condition on the initial data that ensures blow up at infinity of the corresponding global solutions. Moreover, we identify classes of initial data for which this sufficient condition is more general than those previously known (see [1, 2, 3, 4]).

The research is supported by the Science Fund of the UNWE under Grant No. NID NI-19/2025/A.

***K*eywords** Nonlinear wave equation · Logarithmic nonlinearities · Blow up at infinity

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A PERFORMANCE COMPARISON OF E-VALUES AND P-VALUES IN REAL-TIME SEQUENTIAL TESTING

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ABSTRACT

In statistical hypothesis testing, p-values are the conventional measure for assessing evidence against a null hypothesis. However, their reliability is often questioned in scenarios involving multiple hypothesis testing, where e-values offer a promising alternative. This study presents a practical comparison of p-values and e-values within a sequential testing framework. Simulated data, designed to mimic a real-time monitoring scenario, is used to evaluate how both measures behave in detecting an effect, with their performance compared across a range of critical metrics. Findings highlight the key advantages and limitations of each measure.

***K*eywords** p-values · e-values · real-time sequential testing

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BILINEAR MULTIPLIERS ON WEIGHTED ORLICZ SPACES ON LOCALLY COMPACT GROUPS

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ABSTRACT

In this work, we generalize the theory of bilinear multipliers acting on weighted Orlicz spaces from $\mathbf{R}^n$ to locally compact abelian groups G. We focus on describing these bilinear multipliers from the point of view of abstract harmonic analysis. We obtain separate necessary and sufficient conditions for the existence and boundedness of such bilinear multipliers.

***K*eywords** Bilinear multipliers · Weighted Orlicz spaces · Locally compact groups

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A VIETORIS-BASED NUMBER SEQUENCE AND THEIR APPLICATION IN CRYPTOGRAPHY

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ABSTRACT

In this study, one considers an integer sequence associated with the sequence of rational numbers known as the *Vietoris' sequence*. In 1958, L. Vietoris presented a result in [5] concerning the positivity problems of trigonometric sums, wherein this sequence naturally emerged. The first Vietoris' numbers of the sequence are

$$1, \frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{3}{8}, \frac{5}{16}, \frac{5}{16}, \frac{35}{128}, \frac{35}{128}, \frac{63}{256}, \frac{63}{256}, \frac{231}{1024}, \frac{231}{1024}, \cdots,$$

which is related with the sequence A283208 in the On-Line Encyclopedia of Integer Sequences (OEIS) in [4]. This sequence, which will be denoted by $\{v_n\}_{n\geqslant 0}$, is defined as

$$v_n = \frac{1}{2^n}\binom{n}{\lfloor \frac{n}{2} \rfloor}, \qquad n \geqslant 0,$$

where $\lfloor \cdot \rfloor$ is the floor function. It is also well established that the recurrence relation governing this sequence is given by the following expression

$$v_n = \begin{cases} 1, & n = 0 \\ d_n\ v_{n-1}, & n \neq 0 \end{cases} \tag{1}$$

where, for $\sigma(n) = \frac{1+(-1)^n}{2}$,

$$d_n = \frac{n+\sigma(n)}{n+1} = \begin{cases} 1, & n\ even \\ \frac{n}{n+1}, & n\ odd \end{cases}.$$

For further information on Vietoris numbers, see for example [1, 3].
In view of the identity $v_{2n+1} = v_{2n+2}$ for $n \in \mathbb{N}_0$, it is natural to consider the subsequence of Vietoris' sequence with odd index $\{v_{2n+1}\}_{n\geqslant 0}$. In [2] the elements of sequence $\{v_{2n+1}\}_{n\geqslant 0}$ were explicitly represented as follows

$$v_{2n+1} = \frac{a_n}{2^{n+1+m_n}}, \tag{2}$$

where the sequence $\{a_n\}_{n\geqslant 0}$ is the numerators of the subsequence of the Vietoris' number sequence $\{v_{2n+1}\}_{n\geqslant 0}$ and

$$m_n = \left\lfloor \frac{n+1}{2} \right\rfloor + \left\lfloor \frac{n+1}{2^2} \right\rfloor + \cdots + \left\lfloor \frac{n+1}{2^m} \right\rfloor, \quad m \leq \log_2(n+1). \tag{3}$$

The sequence $\{n+m_n\}_{n\geqslant 0}$ is called the minimal exponent integer sequence with respect to 2 and is the sequence A283208 in the OEIS in [4].

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Due to inherent challenges and potential vulnerabilities associated with using rational sequences in cryptographic encoding and decoding processes, one will consider, in this study the sequence of integer odd numbers $\{a_n\}_{n\geqslant 0}$. The first numbers of the sequence $\{a_n\}_{n\geqslant 0}$ are

$$1,\ 3,\ 5,\ 35,\ 63,\ 231,\ \cdots,$$

which are identified as the sequence A001790 in the OEIS in [4]. These numbers emerge as the numerators of the coefficients in the Maclaurin series expansion associated with the corresponding real-valued function $f(x) = \frac{1}{\sqrt{1-x}}$ for $x < 1$.

On account of equation (2), the sequence $\{a_n\}_{n\geqslant 0}$ can be expressed in terms of Vietoris' numbers, by

$$a_n = 2^{n+1+m_n} v_{2n+1}. \tag{4}$$

Furthermore, applying (2) in (1), on obtains the following recurrence relation

$$a_n = \alpha_{m_n} a_{n-1}, \qquad n \in \mathbb{N},$$

where $\alpha_{m_n} = \frac{2n+1}{n+1} 2^{m_n - m_{n-1}}$ and m_n is defined in (3).

In this study, matrices whose elements are derived from the sequence $\{a_n\}_{n\geqslant 0}$ are introduced, along with an analysis of some of their fundamental properties. These matrices are subsequently employed to explore potential applications in cryptography, particularly in the construction of encoding and decoding algorithms.

***K*eywords** Cryptography · Recurrence relation · Vietoris' number

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ICMASE 2025

CRITICAL THINKING AND ARTIFICIAL INTELLIGENCE IN THE CONSTRUCTION OF STATISTICAL KNOWLEDGE

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ABSTRACT

In the context of higher education for engineering students, the integration of artificial intelligence (AI) tools into the teaching of Statistical Methods presents both pedagogical opportunities and challenges. This study describes the design, implementation, and preliminary analysis of a project-based learning experience structured according to the Dreyfus model of skill acquisition (Dreyfus & Dreyfus, 1980). The initiative, implemented in a Statistical Methods course for Informatics Engineering students, aims to enhance students' understanding of fundamental statistical concepts, such as probability theory, random variables, sampling, estimation, and hypothesis testing, while simultaneously fostering their critical engagement with AI-generated content. In parallel, from a research perspective, the project sought to investigate how students construct and consolidate statistical knowledge over time and how they progress through the five stages of the Dreyfus model as they engage with a sequence of five learning tasks. Students were encouraged to use generative AI tools such as ChatGPT to support their resolution of exercises, but were explicitly required to critique the outputs based on mathematical rigour, conceptual accuracy, and clarity. Assessment rubrics, designed in alignment with the Dreyfus developmental levels (novice to expert), focused on three domains: mathematical precision, critical analysis of AI responses, and clarity of the corrected solution. This approach not only promotes metacognition and the development of self-regulated learning strategies but also aligns with current models of inquiry-based learning and knowledge construction in mathematics education (Dreyfus, 2024; Hershkowitz et al., 2001). Furthermore, the project is informed by frameworks that highlight the importance of critical thinking in problem-solving scenarios (Casiraghi & Aragão, 2019), and explores how AI can function not as an authoritative oracle but as a partner in the learning process, one whose output must be challenged, verified, and, when necessary, corrected. This aligns with recent shifts in the role of technology in the mathematics classroom, from passive tool to active agent of dialogue (Laursen & Rasmussen, 2019; Theobald et al., 2020).

Preliminary qualitative feedback suggests an increase in students' awareness of the limitations of AI tools, improvement in their ability to validate statistical reasoning, and a deeper engagement with course content. Students demonstrated the ability to mobilise prior learning from classes and earlier tasks to enhance their construction of statistical knowledge and improve their academic performance.This model provides a replicable pedagogical framework that aligns with both the development of disciplinary expertise and the cultivation of 21st-century skills such as critical thinking and responsible AI usage.

***K*eywords** Statistical Methods · artificial intelligence · engineering education · Dreyfus model · critical thinking

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ANALYZING SMOKING BEHAVIOR WITH GEGENBAUER POLYNOMIAL APPROXIMATION

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ABSTRACT

This study aims to forecast the smoking behaviors of individuals classified into five distinct groups: potential smokers or non-smokers, light smokers, heavy smokers, and those who have either temporarily or permanently quit smoking. The interactions among these groups are represented by a system of five ordinary differential equations, each corresponding to one subgroup. To solve this system, the Gegenbauer polynomial expansion is employed, enabling accurate approximation of the system's behavior over the wide interval [0, 100]. This approach facilitates a comprehensive analysis of the long-term dynamics under four different sets of initial conditions. Since the system does not admit an analytical solution, the numerical results obtained from the Gegenbauer method are validated by comparison with both the classical fourth-order Runge-Kutta method and Mathematica's built-in differential equation solver. The simulation results illustrate the evolution and interaction of the various smoker groups over time.

***K*eywords** Gegenbauer polynomials · Smoking model · Disease modeling

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T-NORMS IN FUZZY LOGIC: A NOVEL PATH TO OPTIMIZATION

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ABSTRACT

T-norms play a pivotal role in fuzzy logic, serving as essential aggregation operators that define conjunctions in fuzzy systems. Their mathematical properties and computational efficiency have made them fundamental in various applications, from decision-making processes to artificial intelligence. This paper explores a novel approach to optimization via T-norms in fuzzy logic, investigating their potential for enhancing performance in complex problem-solving scenarios. By analyzing different classes of T-norms and their influence on fuzzy inference mechanisms, we aim to develop innovative strategies for improving computational accuracy and efficiency. Experimental results demonstrate the advantages of specific T-norm selections in optimization problems, highlighting their capability to refine uncertainty modeling and enhance decision accuracy. This study contributes to the ongoing discourse on fuzzy logic optimization and offers new perspectives on leveraging T-norms for practical and theoretical advancements.

***K*eywords** Fuzzy logic · Optimization · T-norms

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A COMPUTATIONAL METHOD FOR SOLVING COUPLED BURGERS' EQUATION

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ABSTRACT

The coupled viscous Burgers system plays a significant role in modeling a variety of phenomena in applied sciences, particularly in fluid dynamics and nonlinear acoustics. However, obtaining exact analytical solutions for such systems is often difficult and, in many cases, impractical. This study proposes a numerical algorithm based on a linearization technique to solve the coupled Burgers equations. The linearization is derived from the Newton–Raphson method. The presented scheme employs multiquadric radial basis functions (MQ-RBFs) for spatial discretization and the Crank–Nicolson method for time integration. By combining the meshless flexibility of radial basis functions with the stability of the Crank–Nicolson scheme, the method efficiently solves the coupled system. The accuracy and efficiency of the approach are demonstrated through the numerical solution of several benchmark problems from the literature. The novelty of this work lies in the integration of MQ-RBFs with Newton–Raphson-based linearization, enabling the efficient treatment of strong nonlinearities without requiring complex meshing. Moreover, the proposed methodology can be extended to a wide range of nonlinear partial differential equations and systems arising in heat transfer, traffic flow, and reactive transport modeling.

***K*eywords** Linearization Technique · MQ-RBF · Crank-Nicolson · Coupled Burgers' Equation · Newton-Raphson Method

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HICKS CONTRACTION MAPPING IN INTUITIONISTIC FUZZY B-METRIC SPACES

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ABSTRACT

In recent years, fuzzy b-metric spaces that combine b-metric and fuzzy metric structures have been studied by different authors. Hicks introduced the concept of C-contraction in the study of fixed point theory for fuzzy metric spaces. More recently, Romaguera generalized and unified the previous results within the framework of fuzzy b-metric spaces and further extended Hicks's concept to this new setting. In this study, we define intuitionistic fuzzy b-metric spaces in the sense of Romaguera and introduce Hicks-type contraction mappings within these spaces. We also investigate the completeness of intuitionistic fuzzy b-metric spaces. Our findings generalise the classical results in fuzzy b-metric environment and extend the fixed point theorem in fuzzy b-metric environment to the intuitionistic fuzzy context. This study contributes to a broader understanding of fixed point theory in intuitionistic fuzzy environments and aims to provide a fundamental step for further applications.

***K*eywords** Complete · fixed point · hicks contraction · intuitionistic fuzzy b-metric space

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EXPLORING CREATIVITY AND LEARNING: WORKS DEVELOPED BY ELEMENTARY SCHOOL STUDENTS A LONG TIME AGO

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ABSTRACT

Creativity is one of the most striking characteristics of schoolchildren. In primary school, they demonstrate an incredible ability to imagine, innovate and transform the world around them. From vibrant drawings to fantastical stories, creativity is an essential tool for cognitive, emotional and social development. The school environment plays a fundamental role in stimulating this creativity. Teachers who encourage experimentation, curiosity and freedom of expression help students to develop their own ideas without fear of making mistakes. Artistic, musical and playful activities are essential so that children can explore different forms of creation. Moreover, creativity is not restricted to the arts. In solving mathematical problems, in spontaneous play and even in the way they interact with their peers, children constantly demonstrate their ability to think outside the box. Stimulating this ability from an early age is essential in order to form adults who can face challenges with flexibility and originality. That is why it is essential that school and family work together to create an environment rich in stimuli, where each child feels safe to express their imagination. After all, children's creativity is not just a passing phase, but the basis for a future full of discoveries and achievements.

Over the last 40 years, elementary school children in Portugal have been the protagonists of a dynamic educational process, full of innovation, creativity and community involvement. This study is dedicated to the first author's mother who was the teacher and presents a collection of works developed by her elementary school students, highlighting creativity, critical thinking and interdisciplinarity in the school environment. Through activities that include visual arts, creative writing and scientific experimentation, the students demonstrate their capacity for innovation and collaboration. The analysis of the works reveals how work-based learning contributes to the development of children's socio-emotional and cognitive skills, encouraging autonomy and engagement with knowledge. The results show that valuing student work not only strengthens learning, but also contributes to the formation of active and aware citizens. The study reinforces the importance of an education that respects the past, understands the present and prepares for the future.

***K*eywords** Creativity · Primary Education · Creative learning · Teachers

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SOME PROPERTIES ON BIDIMENSIONAL TRIBONACCI NUMBERS

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ABSTRACT

The Tribonacci sequence, denoted by $\{t_n\}_{n\geq 0}$ and catalogued as A000073* in the OEIS [4], offers fertile ground for new and challenging investigations. Studied and popularised by Feinberg [1], this sequence is defined by the three-term linear recurrence

$$t_n = t_{n-1} + t_{n-2} + t_{n-3}, \quad \text{for all integers } n \geq 3,$$

with initial terms $t_0 = 0$ and $t_1 = t_2 = 1$.

In this work, we explore a two-dimensional generalisation of the Tribonacci sequence, referred to as the bidimensional Tribonacci sequence and denoted by $\{T_{m,n}\}_{m,n\geq 0}$. Building upon the classical one-dimensional case, the bidimensional sequence was defined by Pethe [3] through a natural extension of the Tribonacci recurrence to two indices:

$$\begin{aligned} T_{(m,n)} &:= T_{(m-1,n)} + T_{(m-2,n)} + T_{(m-3,n)}, \quad \text{for all } n \text{ and } m \geq 3 \\ T_{(m,n)} &:= T_{(m,n-1)} + T_{(m,n-2)} + T_{(m,n-3)}, \quad \text{for all } m \text{ and } n \geq 3 \end{aligned}$$

with initial conditions involving complex values:

$$\begin{array}{lll} T_{(0,0)} = 0\,, & T_{(1,0)} = 1\,, & T_{(2,0)} = 1\,, \\ T_{(0,1)} = i\,, & T_{(1,1)} = 1+i\,, & T_{(2,1)} = 1+i2\,, \\ T_{(0,2)} = i\,, & T_{(1,2)} = 2+i\,, & T_{(2,2)} = 2+i2\,, \end{array}$$

where $i^2 = -1$ is the imaginary unit.

Our study extends some known results from the classical Tribonacci sequence (see for instance [2]) into the bidimensional context. In particular, we investigate algebraic properties, recurrence structures, and symmetries of this extension and derive new identities. Special attention is given to the role of complex values in the initial conditions and their impact on the emerging structure. This generalisation not only enriches the theoretical landscape surrounding recurrence relations but also provides a framework for further exploration of multidimensional recurrence relations.

Keywords Tribonnaci numbers · Bidimensional Tribonnaci numbers · Bidimensional recurrence relations

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TRANSFORMING MATHEMATICS EDUCATION: LESSON STUDY AS A PATHWAY FOR FUTURE TEACHERS

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ABSTRACT

In an era where the complexities of science, engineering, and societal challenges demand an ever-evolving educational paradigm, the preparation of future teachers must transcend traditional methods. This study explores an innovative approach to teacher education, integrating active and challenging strategies within an interdisciplinary framework to cultivate educators capable of fostering meaningful and dynamic learning environments.

Grounded in the principles of theory-informed lesson study, this research examines the pedagogical development of prospective primary school teachers. By adapting learning study methodologies to the specific context of teacher preparation, the intervention aimed to bridge the gap between theoretical knowledge and practical application. Seventeen participants engaged in collaborative task design, reflective practice, and iterative refinement. This structured engagement facilitated a discernible transition from conventional expository teaching methods towards student-centred, inquiry-driven pedagogical practices, fostering an understanding of mathematics applied in scientific contexts.

The findings underscore the transformative impact of this approach. Participants demonstrated a significant evolution from merely replicating traditional instruction to adeptly designing lessons that demonstrably stimulate engagement, ignite curiosity, and cultivate critical thinking within their future professional practice. Furthermore, the study highlights the crucial role of structured collaborative reflection in enhancing educators' confidence and pedagogical adaptability.

By aligning mathematical instruction with interdisciplinary applications, this research contributes to the broader discourse on education reform, advocating for methodologies that prepare teachers to meet contemporary educational challenges. The implications extend beyond the domain of mathematics education, offering a scalable model for fostering educator development across disciplines. This study serves as a call to action for embracing evidence-based, reflective teaching strategies that empower future teachers to shape the next generation of learners.

***K*eywords** Lesson study · Prospective teachers · Mathematics

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ON THE CONDITIONING OF VANDERMONDE SYSTEMS WITH MOCK-CHEBYSHEV NODES

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ABSTRACT

The polynomial interpolation problem defined on a set of distinct nodes and represented in a monomial basis results in a linear system of equations with a Vandermonde matrix. A difficulty with a Vandermonde matrix is that when constructed with real nodes, it is generally quite ill-conditioned [1], even for moderately low degrees. This ill-conditioning may vary notably depending on the distribution of the points. The general recommendation is to use the highly non-uniform Chebyshev nodes, but the problem remains when experimental data is available only at equally spaced points. In such cases, polynomial interpolation becomes unreliable due to the Runge phenomenon and is also numerically ill-conditioned. To address these challenges, an effective strategy is to select mock-Chebyshev points from a dense set of uniformly spaced nodes, thereby capturing the favorable interpolation properties of Chebyshev nodes [2, 3, 4].

In this study, we investigate the condition number of the generalized Vandermonde matrix and demonstrate that it can be reduced by employing mock-Chebyshev nodes, similarly to the case of Chebyshev–Lobatto nodes.

***K*eywords** Vandermonde matrix · Condition numbers · Mock-Chebyshev nodes

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MATHEMATICS AND ITS APPLICATIONS IN FORENSICS: CASE STUDIES AND ANALYTICAL APPROACHES

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ABSTRACT

Aim:

This paper aims to critically review the role of mathematics in forensic science, emphasizing how quantitative methods enhance the analysis, interpretation, and validation of forensic evidence. It explores statistical, geometrical, and computational approaches used across diverse forensic disciplines and illustrates these through landmark case studies.

Methodology: A comprehensive literature review was conducted focusing on forensic applications of mathematical techniques such as statistical modeling, Bayesian inference, geometric morphometrics, and machine learning. Selected case studies— including DNA profiling in the Colin Pitchfork case and bite mark analysis in the Ted Bundy investigation—were analyzed to demonstrate practical applications. Emerging computational methods and their integration into forensic workflows were also examined.

Results:

Mathematical methodologies provide objective frameworks that increase the reliability and reproducibility of forensic analyses. Statistical models quantify evidentiary strength via likelihood ratios and probabilistic reasoning. Geometric and pattern analysis facilitate accurate biometric comparisons, while machine learning enhances automated evidence classification. Case studies confirm that mathematical rigor significantly contributes to successful forensic investigations and judicial outcomes.

Conclusion:

The integration of advanced mathematical tools is indispensable for modern forensic science, improving the precision and transparency of evidence evaluation. Continued interdisciplinary collaboration and methodological innovation are essential to address current challenges and fully leverage mathematical techniques in forensic practice.

***K*eywords** Forensic science · Mathematics · Statistical modeling · Bayesian interference · geometric morphometrics · machine learning · forensic evidence

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OPTIMAL SOLUTION METHODS FOR ADJOINT PROBLEMS IN WEIGHTED HARDY CLASSES

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ABSTRACT

This work examines a boundary value problem within the theory of analytic functions in the unit disk:

$$F^+(\tau) + G(\tau) \cdot F^-(\tau) = 0, \quad \|\tau\| = 1.$$

Here, $G(\tau)$ is a given function defined on the unit circle, while $F^{\pm}(z)$ represent the boundary values of functions analytic inside and outside the unit disk, respectively. The problem has been studied in Hilbert spaces (H^p), with methods developed to solve it under more general classes of $G(\tau)$. A.P. Soldatov extended the analysis by considering solutions in weighted Hilbert spaces, introducing additional complexity and generality. Such boundary value problems are closely related to Riemann–Hilbert equations, singular integral equations, and operator theory.

Keywords Biorthogonal system · Bassel basis · Piecewise Hölder functions

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ICMASE 2025

THE HOMOTOPY PERTURBATION METHOD AND NON-LOCAL SOLUTION OF INTEGRO-DIFFERENTIAL EQUATION

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ABSTRACT

Many researchers have focused on integral-differential equations due to their pivotal role in engineering, mathematical modeling, and physical phenomena, and how to solve them under local conditions. These equations have been addressed using various techniques, with some turning to transforming the systems to be solved into a system of integral equations, which can be solved using semi-analytical or numerical methods. This depends on the type of kernel used. However, in this research, the advanced-phase equation will be studied under non-local conditions. This type of modern study enables researchers to identify the strengths and weaknesses of a material before using it, as it reveals the genes and properties of the material being used. Therefore, in this work, we will attempt to study a non-linear integro-differential equation with a continuous kernel using analytical methods. The problem will also be studied by comparing solutions under local conditions with solutions under non-local conditions and studying the resulting error in each case.In this study we will consider the the phase-lag nonlinear integro-differential equation

$$\frac{\partial}{\partial t}\left[\Psi\left(x,\, t+p\right)-f(x,t)\right]=\lambda\, h(t)\int_0^1 k\left(x,\, y\right)\Psi^{\alpha}\left(y,\, t\right)\, dy,\quad \alpha=1,2,3,\ldots,N.(1) \qquad (1)$$

under the non-local conditions

$$\Psi\left(x,\, p\right)=v_1\left(x,\, p\right)+v_2\left(x,\, t\right),\Psi\left(x,\, 0\right)=v_3(x)+v_2(x,\, t),$$

$$\frac{\partial\,\Psi\left(x,\, 0\right)}{\partial\, t}=v_4\left(x\right)+\frac{\partial}{\partial\, t}\,v_2\left(x,\, t\right).$$

Here, $f\left(x,t\right)$ is a free term, $h\left(t\right)$ is a continuous function of time, λ is a parameter and $k\left(x,y\right)$ is a known continuous function that represents the kernel of position. The unknown function that has to be determined is $\Psi\left(u,\, t\right)$. Where, $\Psi^{\alpha}\left(u,\, t\right)$ represents a nonlinear term in the equation. Under considering Taylor's expansion,

$$\Psi(x,\, t+p)=\Psi\left(x,\, t\right)+\frac{p}{1!}\,\frac{\partial\,\Psi\left(x,\, t\right)}{\partial\, t}+\frac{p^2}{2!}\,\frac{\partial^2\,\Psi\left(x,\, t\right)}{\partial\, t^2}+\ldots,$$

A nonlinear mixed integral equation of the second kind with a continuous kernel will be created from equation (1). In order to solve the resulting integral equation, the Homotopy approach will be used in this search. We shall talk about several significant theorems. Applications' numerical solutions will be offered to confirm the effectiveness of the strategies discussed.

***K*eywords** Integro partial differential equation · Mixed integral equation · phase-lag problem

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ICMASE 2025

THIRD-ORDER NICKEL FIBONACCI QUATERNIONS

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ABSTRACT

In this presentation, we first define the third-order Nickel Fibonacci quaternions, based on the framework of third-order Nickel Fibonacci numbers. We then establish several well-known identities for these quaternions, including Binet's formula and the generating function. Furthermore, we introduce the generalized third-order Nickel Fibonacci quaternions and present their corresponding matrix representations. The main obstacles in the study of quaternion matrices, as expected come from the non-commutative multiplication of quaternions. Since the classical determinant is not suitable for quaternion matrices, we use the q-determinant and establish the related matrix identities.

***K*eywords** Nickel Fibonacci Numbers · Binet's identity · q-determinant

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NORMAL APPROXIMATIONS FOR THE ERGODIC DISTRIBUTION OF A SEMI-MARKOVIAN INVENTORY MODEL OF TYPE (S,S)

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ABSTRACT

In practical applications, the most commonly utilized models for random recurrent events are renewal processes, renewal reward processes, random walk processes and their modifications. These stochastic models serve as foundational tools for modeling and solving a wide range of problems across various disciplines. For instance, they are instrumental in analyzing purchasing behavior following warranty expiration in warranty analysis, modeling inter-arrival times between demands in inventory systems, planning logistics in supply chain management and analyzing sequential risks in risk theory. The extensive literature on these processes includes a wealth of theoretical results and properties, reflecting their foundational role in applied probability and operations research. Despite the importance and applicability of these models, deriving closed-form expressions for their ergodic distributions often presents significant analytical difficulties. In real-world applications, there is often a need for practical methods that can bypass these complexities while still yielding accurate results.

This paper presents three different approximations to the ergodic distribution of a renewal reward process X(t) describing a semi-Markovian inventory model of type (s,S), where the underlying demand random variables follow a Weibull distribution. While the Weibull distribution flexibly captures various real-world behaviors, its associated renewal function lacks a closed-form expression, posing analytical challenges. To address this, we make use of three approximation formulas for the Weibull renewal function based on normal approximations of the Weibull distribution, as proposed in the study of Cui and Xie [1]. These approximations provide tractable expressions for the renewal function of large Weibull shape parameter, which are then used to derive approximations for the ergodic distribution of the renewal reward process X(t). We used easily applicable methods that offer fast and accurate approximations while maintaining interpretability. Our results do not rely on asymptotic expansions, avoiding remainder terms whose convergence properties are often unclear. Numerical results are presented to compare the accuracy of the three approximations for different Weibull shape parameters.

***K*eywords** Semi-Markovian inventory model · Ergodic distribution · Weibull renewal function · Normal approximation

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ON SPATIAL SEDENIONIC CURVES

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ABSTRACT

In this study, we define the vector product of certain vectors in the Euclidean space $\mathbb{R}^{15}$. Using this definition, we reformulate the sedenion product and explore its properties within $\mathbb{R}^{15}$. We then derive Serret–Frenet formulas for spatial sedenionic curves and construct a G_2-frame adapted to such curves. In the final section, we provide several MATLAB codes developed for sedenion-related computations. These implementations support numerical applications and facilitate theoretical investigations involving sedenions.

***K*eywords** Sedenions · G_2- Frame · Vector Product

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ICMASE 2025

SPECTRAL ANALYSIS OF REACTIONDIFFUSION SYSTEMS VIA PINNS

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ABSTRACT

Reactiondiffusion systems are widely used to describe dynamic behaviors and pattern formation in fields such as physics, chemistry, and biology. A key part of understanding these systems is analyzing the stability of steady-state solutions, which typically involves solving an eigenvalue problem derived from a linearization of the governing equations. However, exact solutions to such problems are often out of reachparticularly when the system includes multiple components or when the operators involved are not self-adjoint. In these cases, researchers turn to numerical methods as the most practical means of performing spectral analysis and exploring the systems behavior. In this study, we explore the use of Physics-Informed Neural Networks (PINNs) to solve these types of eigenvalue problems. PINNs offer a data-free, physics-driven way to learn solutions of partial differential equations by incorporating the governing laws directly into the training process [1, 2]. We apply this method to compute multiple eigenpairs in both single- and multi-component reaction-diffusion systems, including scenarios that are especially difficult for conventional approachessuch as nonself-adjoint systems. To demonstrate the method, we focus on two benchmark models: the ZeldovichFrankKamenetsky (ZFK) equation [3], representing a single-component and self-adjoint case, and the FitzHughNagumo (FHN) system [4], which introduces a two-component, nonself-adjoint structure. Our PINN framework enforces critical physical properties like biorthonormality between left and right eigenfunctions and spectral ordering within the loss function itself. As a result, it produces eigenvalue and eigenfunction estimates that align well with results obtained through standard numerical simulations. This work highlights the potential of PINNs as a robust and adaptable approach to spectral analysis in complex reactiondiffusion systems.

***K*eywords** PINN · deep learning · reactiondiffusion · eigenvalue problems · FitzHughNagumo · ZeldovichFrankKamenetsky · adjoint

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BAYESIAN PHYSICS-INFORMED NEURAL NETWORKS FOR CURVE ESTIMATIONS IN EXCITABLE MEDIA

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ABSTRACT

Strengthduration curves characterize the relationship between stimulus duration and the minimal strength required to trigger excitation, offering critical insight into the excitability and safety margins of excitable systems [1]. While these curves can, in principle, be derived analytically, such derivations are typically intractable except in a few highly idealized cases. As a result, constructing them often relies on direct numerical simulations, which become computationally demanding in multi-component systems or when the governing operators are not self-adjoint. To address these challenges, we introduce a Bayesian physics-informed neural network framework as an alternative approach for learning the entire strengthduration profile. Our method embeds the reactiondiffusion dynamics directly into the networks loss function, ensuring adherence to known biophysical laws, and treats the critical threshold curve as a hidden dependency learned during training. We employ Hamiltonian Monte Carlo to sample the posterior distribution of network [2] and threshold parameters, yielding wellcalibrated credible intervals that outperform variational and dropoutbased uncertainty estimates. A single Bayesian PINN model yields a smooth, continuous threshold curve with principled uncertainty estimates, enabling efficient exploration of excitability across parameter regimes. By combining the data efficiency and interpretability of physics-informed neural networks with the rigor of Bayesian inference [3, 4], this framework enables rapid discovery while providing robust uncertainty quantification, offering a flexible tool for investigating threshold phenomena in reactiondiffusion systems and related contexts. Overall, the Bayesian PINN achieves accurate, continuous estimates of strengthduration relationships and furnishes well-calibrated uncertainty bands, making it a reliable and interpretable tool for probing threshold phenomena in complex excitable systems.

***K*eywords** strengthduration · Bayesian · PINN · Monte Carlo · reaction-diffusion · threshold · credible interval

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BRIDGING REINTEGRATION AND EQUITY THROUGH LOGIC AND AI: TEACHING FORMER COMBATANTS IN COLOMBIA

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ABSTRACT

This presentation explores the use of mathematical logic and computational thinking as educational tools to support the reintegration of former combatants from the Colombian armed conflict. Building on earlier work in digital literacy (Torres et al., 2014), it addresses the social and economic challenges this population faces, including stigmatization and limited job opportunities (Zapata et al., 2021). The proposed educational model integrates digitalization and Artificial Intelligence (AI) to enhance logical reasoning and problem-solving skills (Behnamnia et al., 2025), using pedagogical prototypes and mathematics labs (Torres & Martínez, 2015).

A study involving over 30 ex-combatants and 260 professionals from the Agency for Reintegration and Normalization (ARN) revealed a digital divide not of access, but of usage. Although most participants owned smartphones and had internet access, few used these tools for entrepreneurship or education. The intervention included tablets, multimedia content, and a learning platform, achieving measurable improvements in digital competence.

The approach promotes logical and computational thinking as essential 21st-century skills (More Valencia et al., 2022; Soufan et al., 2023), offering personalized and gamified learning supported by AI (Jadhav et al., 2025). This not only benefits cognitive development but also fosters confidence and employability. Mathematics education is framed within inclusive and flexible strategies, requiring a transformation in teacher training. This includes Pedagogical Content Knowledge (Martín et al., 2023) and the application of Didactic Suitability (Barboza & Castro, 2023) to design relevant and equitable instruction.

By fostering logical reasoning and computational literacy, this initiative equips former combatants with tools to navigate an increasingly digital world, enabling them to access education, employment, and civic participation. More than a technical training program, it represents a transformative educational pathway that acknowledges their past, leverages their discipline and resilience, and repositions them as active and capable members of society. The integration of digital and AI-based pedagogies not only bridges knowledge gaps but also supports a broader agenda of peacebuilding and social justice in post-conflict Colombia.

***K*eywords** ex-combatants, mathematical logic, digital literacy, artificial intelligence, reintegration, educational equity

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ON THE GENERALIZATION OF THE CHLODOWSKY-TYPE SZÁSZ OPERATORS INCLUDING POLYNOMIALS

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ABSTRACT

In this paper, we present a new approach by using Chlodowsky-type Szász operators combining special polynomials. The first part of our research focuses on determining the convergence rate of this newly constructed operator. In particular, we analyze how quickly and accurately these operators converge to given functions. Then, several approximation results are derived that provide insight into the effectiveness of the operators on different classes of functions. In addition, we present error estimates based on the weighted modulus of continuity, supported by tabulated numerical data showing the accuracy and stability of the operators in function approximation. In the last section, a Voronovskaya-type theorem is established that characterizes the asymptotic behavior and derivative properties of the operators as the parameter goes to infinity.

Keywords Approximation theory · Chlodowsky-type Szász operators · Weighted modulus of continuity

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ICMASE 2025

DIFFERENTIAL OPERATOR-BASED IMPUTATION: A ROBUST METHOD FOR INCOMPLETE DATA ANALYSIS

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ABSTRACT

When only some data points are available in a dataset, imputation of the approximate values at unknown points often becomes essential to maintain the integrity and usability of the data. This need becomes especially critical in real-world applications where unobserved or missing data are prevalent and can severely impact the performance of statistical analyses and machine learning models. To address this issue, this study introduces a novel imputation technique that reconstructs missing data points using only the known entries, without requiring any prior assumption about the underlying data distribution or functional form.

The method is based on three fundamental equations derived using approximate differential operators. These equations consist of the central difference and the three-point forward and backward approximate differential operators. The value at an unknown point is computed using these equations by incorporating the neighboring data points to its left and right.

To rigorously assess the accuracy, stability, and generalization capacity of the proposed approach, synthetic datasets are generated from various analytical functions that exhibit different behaviors within a fixed domain. These datasets simulate distinct imputation scenarios under controlled conditions, providing a robust testing environment. Comparative analyses with state-of-the-art imputation methods are performed using standard error metrics, including Mean Absolute Error (MAE), Mean Relative Error (MRE), and Root Mean Square Error (RMSE).

Experimental results show that the proposed method consistently delivers high-precision imputations, even when the proportion of missing data is considerable. The local nature of the computation, which is based solely on the nearest neighbors, enhances both efficiency and interpretability. Furthermore, the use of approximate differential operators contributes to the numerical reliability and transparency of the method, making it particularly well suited for critical applications involving incomplete or noisy data.

Keywords Missing Data Imputation · Approximate Differential Operators · Local Estimation

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ICMASE 2025

THE L_p EXTREMAL POLYNOMIALS CORRESPONDING TO POLYNOMIAL SZEGŐ MEASURE

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ABSTRACT

For a the polynomial Szegő class measure μ on the unit circle $\mathbb{T}$ in the complex plan, with $d\mu = \mu'_{ac}dm + d\mu_s$, where μ_{ac} is the absolutely continuous part of μ and $d\mu_s$ is singular and,

$$\int_{\mathbb{T}} p(t) \log \mu'_{ac}(t) dm(t) > -\infty. \tag{1}$$

We define the extremal polynomials $(1 < p < \infty)$ corresponding to polynomial Szegő measure,there are many intersting problems about extremal polynomials . The most important ones are their asymptotics and zero distribitons . For $p = 2$, the $L_p(\mu)$ extremal polynomials are exactly the orthogonal polynomials associated to the measure μ.

***K*eywords** Extremal polynomials · Szegő condition · polynomial Szegő condition.

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THE ASYMPTOTICS FOR L_p EXTREMAL POLYNOMIALS CORRESPONDING TO POLYNOMIAL SZEGŐ MEASURE

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ABSTRACT

For a the polynomial Szegő class measure μ on the unit circle $\mathbb{T}$ in the complex plan, with $d\mu = \mu'_{ac}dm + d\mu_s$, where μ_{ac} is the absolutely continuous part of μ and $d\mu_s$ is singular and,

$$\int_{\mathbb{T}} p(t) \log \mu'_{ac}(t) dm(t) > -\infty. \tag{1}$$

We investigate the asymptotic behavior of $L_p(\mu)$ extremal polynomials $(1 < p < \infty)$ corresponding to polynomial Szegő measure, our main result is the explicit strong asymptotic formulas for the $L_p(\mu)$ extremal polynomials.

Keywords Extremal polynomials · asymptotics for L_p extremal polynomials · polynomial Szegő condition.

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OR/MS EDUCATION: EUROPEAN STEM TEACHERS' PERCEPTIONS ON STUDENTS ENROLMENT AND TRANSITION TO LABOUR MARKET

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ABSTRACT

The present work focus on the perceptions of the Science, Technology, Engineering, and Mathematics (STEM) teachers concerning two specific topics: (A) the enrolment of students; and (B) the transition to the labour market. For that, a statistical analysis that was addressing the lecturing modules of the survey developed within the EURO-European Association of Operational Research Societies 's umbrella [1] is now treated by focusing Computer Science, Engineering, and Mathematics' respondents. A description of the STEM teachers' answers, to each question, is presented and analysed in terms of Positive and Negative perceptions, as well as Non-existence or Lacking sufficient information (NE) on the subject. And the type of OR/MS lecturing modules existent in the respondents´ HEI is also taken into account. Regarding the enrolment of students (topic A), although there is a positive perception about the existence of plans to promote it, the opposite occurs relatively to the existence of joint work projects with pre-university teachers. Concerning the existence of an interlocutor to establish contacts with primary/secondary schools, levelled opinions could be observed, being that approximately a third of the respondents' report Non-existence or Lacking sufficient information on the topic. Additionally, 44% of the STEM teachers have a positive opinion of the assessment activities related to the enrolment of the students. Concerning the specific OR/MS queries, not only a negative opinion was stated by more than 30% of the respondents, but also more than 55% of them refer Non-existence or Lacking sufficient information on the subject. Concerning the transition onto labour market (topic B), an almost positive perception (47% of the responses) can be observed in both the promotion of the transition of graduates onto labour market and the assessment of activities related to labour market, while, levelled Positive/Negative opinions could be perceived regarding the existence of an interlocutor to establish contacts with labour market organizations. With reference to the specific OR/MS queries, a negative perception could be noticed and 28% of the respondents indicated that do not know or do not exist specific promotion of entries into labour market in the OR/MS field. In addition, the relations between the STEM teachers' perceptions and the types of OR/MS lecturing modules (bachelors/masters, majors/minors, or just courses) in their own institutions are also examined. No relationship between the types of OR/MS modules and almost all the questions can be perceived; there is only one single exception, concerning the general HEI's transition of graduates onto labour market; but even in this case only a weak relationship could be observed. The results are detailed and a comparison analysis is developed; it can be concluded that more attention should be payed to these two important topics — enrolment of students and transition onto labour market— and thus more work has still to be done in this area of knowledge.

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***K*eywords** Education · OR/MS modules · enrolment · transition into labour market

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ICMASE 2025

NEW DIFFERENCE SETS CONSTRUCTED VIA THE CYCLIC CLASSES METHOD

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ABSTRACT

Difference sets are fundamental objects in combinatorial design theory due to their applications in error-correcting codes, finite geometries, and cryptography. A subset $D \subseteq G$ of a finite group G of order v is called a (v, k, λ)-difference set if every non-identity element of G can be expressed as a difference $d - d'$, with $d, d' \in D$, in exactly λ ways. Constructing such sets remains a central problem in algebraic combinatorics.

In this study, we develop and apply a method called the *Cyclic Classes Method (CCM)* to generate difference sets in additive groups of finite fields. Let q be a prime power and e a positive integer such that $e \mid (q-1)$. The multiplicative group $\mathbb{F}_q^*$ is partitioned into e cyclotomic classes of order e, denoted $C_e^0, C_e^1, \ldots, C_e^{e-1}$. The CCM strategically combines these classes to form candidate subsets in the additive group $(\mathbb{F}_q, +)$, and verifies the difference set condition using group ring identities and character sums.

Using this approach, we construct new families of difference sets with parameters not previously reported in the literature. The method also allows for algorithmic generalization and automated verification using MATLAB, offering a powerful tool for exploring large search spaces in $\mathbb{Z}_q$.

Our results contribute to the existing catalogue of known difference sets and highlight the utility of cyclotomic structures in finite fields. The study includes theoretical proofs supported by computational evidence and offers insights into future directions for algebraic and algorithmic design theory.

Keywords difference sets · cyclotomic classes · finite fields

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ICMASE 2025

HOMOTOPIC CONTRACTION MAPPINGS

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ABSTRACT

In this study, homotopic contraction mappings studied by Frigon in 1991 were investigated. Kannan type homotopic contraction mapping and Chatterjea type homotopic contraction mapping are defined and some fixed point results are obtained.

***K*eywords** Homotopy · Contraction Mapping · Fixed point

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ICMASE 2025

ON THE GEOMETRY OF SLANT SUBMANIFOLDS IN (α, p)-GOLDEN RIEMANNIAN MANIFOLDS

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ABSTRACT

The notion of an almost (α, p)-golden structure, recently introduced by Hretcanu and Crasmareanu [1], extends the classical golden and metallic structures through a unified polynomial framework on Riemannian manifolds. This construction generalizes earlier works on golden structures [2] and metallic structures [3], and provides a natural setting for studying geometric properties of submanifolds. In their foundational paper, the authors examined invariant submanifolds in this context [1, 6]. Building upon this framework, we consider the geometry of slant submanifolds, which are a natural extension of invariant and anti-invariant cases in submanifold theory [5]. Our results aim to contribute to the development of submanifold geometry within generalized polynomial-type metric structures.

Keywords α-structure · (α, p)-structure · (α, p)-golden manifold · slant submanifold

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NUMERICAL SOLUTION OF DELAY DIFFERENTIAL EQUATION WITH BOUNDARY LAYER

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ABSTRACT

In this work, we consider a boundary value problem for the singularly perturbed convection diffusion problem for a type of delay differential equation. Differential equations with a small positive parameter multiplied by the highest derivative term are said to be singularly perturbed differential equations. Generally, the solutions of such equation exhibit multiscale phenomena. Within certain thin sub-regions of the domain, the scale of some derivatives is significantly larger than other derivatives. These thin regions where rapid change occurs are called boundary or interior layers. These equations appear frequently in mathematical problems in the sciences and engineering. For example, high Reynolds number flows in the fluid dynamics, electrical networks, reaction-diffusion processes, control theory, the equations governing flow in porous media, the drift-diffusion equation of semiconductor devices are modelled by these equations [3, 5].

Generally, solutions to such problems often involve a boundary or initial layer. The classical discretization methods to solve singularly perturbed problems do not work well and fail to give analytical solution when tends to zero. For this reason, it is necessary to develop suitable numerical methods that are uniformly convergent with respect to solve this type of problems [2, 3]. If the singularly perturbed differential equations also contain the delay term, they are called the singularly perturbed delay differential equations. Lange and Miura investigated the asymptotic properties of the boundary value problem for singularly perturbed delay differential equations [4]. We are motivated by this paper for the numerical solution of our work. There are few studies in the literature using different numerical methods to solve these equations [1, 2].

For the numerical solution of the problem, we present a fitted difference scheme on a layer adapted mesh and analyze the error estimates. We show that the scheme is first-order convergent with respect to the perturbation parameter in the discrete maximum norm. Furthermore, we give the numerical experiments which support the theoretical results.

Keywords Delay differential equation · Boundary layer · Finite difference method · Uniform convergence

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ICMASE 2025

ON THE METHOD OF TRANSFORMING A NONLOCAL BOUNDARY VALUE PROBLEM INTO A CAUCHY PROBLEM FOR A SECOND ORDER ORDINARY DIFFERENTIAL EQUATION

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ABSTRACT

Finding solutions to problems with non-local boundary condition, even for in the simple of ordinary differential equations or systems of equations, requires special research. In this work we will study the following equation

$$\ell[y] \equiv y''(x) + ay'(x) + by(x) = 0, \quad 0 < x < 1, \tag{1}$$

to the subject to linearly independent nonlocal boundary conditions with constant coefficients

$$L[u] \equiv \alpha_{k0}y(0) + \alpha_{k1}y(1) + \beta_{k0}y'(0) + \beta_{k1}y'(1) = \gamma_k, \quad (k = 1, 2). \tag{2}$$

Here a, b, α_{kj}, β_{kj} and γ_k, $(k = 1, 2; j = 0, 1)$ are given real constants.
For problem (1), (2) the following as called a basic relation

$$y'(0)Z(-\xi) - y(0)Z'(-\xi) - y'(1)Z(1-\xi) + y(1)Z'(1-\xi) + ay(0)Z(-\xi) - ay(1)Z(1-\xi) =$$
$$= \begin{cases} y(\xi), & \xi \in (0,1), \\ \frac{1}{2}y(\xi), & \xi = 0,\ \xi = 1 \end{cases} \tag{3}$$

is found. $Z(\xi)$ denotes the fundamental solution of the adjoint equation corresponding to (1) in the sense of Lagrange. From (3) we get two necessary conditions as follows

$$\frac{1}{2}y(0) = y'(0)Z(0) - y(0)Z'(0) - y'(1)Z(1) + y(1)Z'(1) + ay(0)Z(0) - ay(1)Z(1), \tag{4}$$

$$\frac{1}{2}y'(0) = y'(1)Z'(1) - y'(0)Z'(0) + by(1)Z(1) - by(0)Z'(0) \tag{5}$$

The following theorems are proved:

Theorem 1. If a and b are given real constants, then any solution of equation (1) satisfies to linear independent necessary conditions (4) and (5).

Theorem 2. Let us assume that a, b, α_{kj}, β_{kj} and γ_k, $(k = 1, 2; j = 0, 1)$ are given real constants and the boundary conditions (2) are linearly independent. Under certain conditions, problem (1), (2) is reduced to the Cauchy problem.

Keywords Nonlocal boundary condition · First and second basic relation · Necessary conditions

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ICMASE 2025

EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR IMPULSIVE ψ-CAPUTO FRACTIONAL BOUNDARY VALUE PROBLEMS

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ABSTRACT

Modern complex systems across diverse scientific and engineering domains require mathematical frameworks that capture both memory effects and sudden disruptions simultaneously. Classical differential equation approaches, while successful for many applications, have limitations when modeling systems that exhibit both hereditary properties and abrupt state changes. This recognition has motivated the development of mathematical frameworks combining fractional calculus with impulsive dynamics, creating powerful tools for modeling the memory dependence and discontinuous behaviors characteristic of many real-world phenomena.

The $\psi-$Caputo fractional derivative represents a significant advancement in fractional calculus by generalizing the classical Caputo operator through the introduction of a kernel function ψ. This generalization enables application-specific adaptation of memory characteristics, providing flexibility that has proven valuable across applications from biomedical engineering and epidemiology to finance and ecological management. The ψ-Caputo framework allows researchers to tailor memory effects to specific system behaviors, significantly expanding the modeling capabilities of fractional differential equations.

While substantial progress has been made for orders $\alpha \in (0, 1]$, higher-order systems with $\alpha \in (1, 2]$ present additional mathematical challenges that require sophisticated analytical approaches. The development of rigorous theory for these systems is particularly important because many physical phenomena naturally require $\alpha > 1$ to capture both strong memory and acceleration effects. Fractional diffusion-wave equations and vibrating systems with hereditary properties exemplify applications that inherently involve momentum effects requiring fractional orders greater than unity for proper mathematical representation.

This work establishes existence and uniqueness results for impulsive $\psi-$Caputo fractional differential equations of order $1 < \alpha \leq 2$ using fixed point theorems. Under appropriate conditions, we prove that these differential equations possess unique solutions. These theoretical results provide essential analytical foundations that complement the computational methods required for solving non-local fractional operators. Our contribution advances the mathematical theory while enabling enhanced modeling capabilities for systems requiring both complex memory effects and sudden state changes, with the flexible $\psi-$Caputo framework supporting application-specific adaptations across diverse scientific domains.

***K*eywords** Impulsive · ψ-Caputo Fractional Boundary Value Problem · Fractional Calculus · Fixed Point Theorem

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ICMASE 2025

A Complexity Analysis of an Infinity Norm Neighborhood Predictor-Corrector Linear Programming Version Algorithm

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Abstract

Predictor-Corrector methods are one of the most studied variants of interior-point methods, being Mehrotra's predictor-corrector algorithm for Linear Programming [2] one of the most successfully implemented, both in research works and commercial packages.
One way to advance in the investigation on interior-point methods is to obtain more efficient algorithms. A manner of achieving this goal is to search for methods with better complexity results; this is, methods with a lower upper bound on the time needed by an algorithm to solve a given instance. Several researches have devoted their attention to the study of the complexity of Linear Programming algorithms; namely, the works of Zhang e al. [3], [4], [5], [6], Dexter et al. [7] and Benhadid et al. [8], that proved the polynomial complexity of some Linear Programming interior-point method variants.
In [1], Bastos and Paixão presented some Linear Programming interior point approaches, directly targeted to some linear allocation problems, specifically transportation and assignment, where the special structure of the constraint matrix of these problems is used to simplify the computations. The current work focus on one of these methodologies, namely the primal-dual predictor-corrector one. In particular, a feasible variant of this algorithm, originally proposed for structured problems such as transportation and assignment problems, is specifically adapted to use the infinity norm neighborhood and, then, analyzed.
The infinity norm is an alternative to the 2-norm when computing the used neighborhood, as it is more scalable and allows for greater flexibility in individual component deviations, particularly regarding computational efficiency and local accuracy, while the 2-norm can be excessively restrictive in practice. Additionally, the infinity norm often defines a broader neighborhood, allowing the algorithm to take longer steps without violating feasibility constraints, and, thus, progressing faster in the initial iterations, particularly when still being far from optimality.
In this work, a complete theoretical analysis, that includes the derivation of step-size bounds, duality gap reduction, and the complexity bound is presented. It is demonstrated that the algorithm has polynomial complexity and Q-linear convergence is proved. To achieve the main result, some additional lemmas and propositions were developed and demonstrated. This process allowed to prove that the algorithm with the infinity norm neighborhood has iteration complexity of $\mathcal{O}(n^{1/2}\log(\frac{1}{\varepsilon}))$.

***K*eywords** Feasible Predictor-Corrector Algorithm · Infinity Norm Neighborhood · Polynomial Complexity

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ICMASE 2025

A New Aspect to Octonions

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Abstract

In this study, we will examine octonions as a new number system from a different perspective. By changing the roles of the units of real octonions $e_0, e_1, e_2, e_3, ..., e_7$ we will give the octonions constructed in a new form and their basic properties. Moreover, the basic algebraic properties, addition and multiplication operations and their differences from other octonion structures will be examined and their geometric properties will be given. In addition, it will be discussed what further studies can be built and worked on this new number system and from which directions it can be approached.

Keywords Octonions · Galilean Geometry

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THE LINEAR CANONICAL TRANSFORMS ASSOCIATED WITH THE DUNKL–TYPE OPERATOR AND ITS APPLICATIONS

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ABSTRACT

In this paper, we introduce the canonical Fourier transform associated with the Dunkl-type operator and study some of its important properties. We study the generalized translation operator and convolution product associated with this transform and present some of its basic properties. Next, at the heart of our investigation, we introduce and study the canonical wavelet transform associated with the Dunkl-type operator. Nonetheless, special attention is paid to the applications of these new transforms.

***K*eywords** Generalized canonical Fourier transform · generalized canonical wavelet transform · Dunkl-type operator · Jacobi–Dunkl transform · Chébli–Trimèche hypergroup · translation operator · convolution product

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VI International Conference on Mathematics and its Applications in Science and Engineering
(ICMASE 2025)

ICMASE 2025

VIBRATION-BASED REMOVAL OF VARIOUS MARTIAN DUST SIMULANTS FROM PHOTOYOLTAIC MODULES IN A LOW-COST SIMULATED MARTIAN ENVIRONMENT

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ABSTRACT

Solar energy is a crucial power source on Earth and for space exploration rovers. However, dust deposition can reduce solar panel power output by more than 50% on Earth [l], an average of 0.2% per Sol [2], and over 80% under simulated lunar conditions [3]. Traditional mitigation strategies such as electrostatic cleaning and coatings lose effectiveness under Martian conditions of low humidity and abrasive dust [4,5]. However, vibrational dust removal shows promise if the duration of vibration is optimized to avoid sensor damage [6]. Typical Martian simulation chambers fail to account for different gravitational conditions on both Mars and Earth. Since Martian gravity is a third of Earth's gravity, previous studies conducted by NASA have used walnut shell dust as a low-density simulant [7]. To investigate the performance of a vibrational system under realistic conditions, we designed a cost-effective experimental chamber simulating the Martian environment by combining vacuum conditions with rotation. Using this method, we increased gravitational acceleration by adding centripetal acceleration through rotation. While Martian gravity is lower than that of Earth, low-gravity simulation methods are inaccessible. Utilizing this increase in gravity, an exponential decay function was used to predict performance at Martian conditions.This device allowed the study of dust removal efficiency on solar panels at varying tilt angles $(0°, 5°, 10°)$ and simulated gravity levels $(25, 35, 45\ RPM)$. True Martian dust is toxic and would behave differently on Earth compared to on Mars. Glass microbeads were used in this study to stand in for true Martian dust since they have the same density. This simulant underwent rotation in this chamber such that performance at a lower gravitational !eve! could be extrapolated. Walnut shell dust was the control simulant as it did not require rotation, as is already approximately a third of the density of true Martian dust. Restoration times required to recover 25% of original panel power output were recorded, alongside measurements of the power consumed by the dust removal system. Supporting the experimental work, a mathematical force-balance model was developed to describe dust partide acceleration resulting from the combined effects of gravitational, vibrational, frictional, and centripetal forces. The centripetal acceleration on a dust partide a was calculated in terms ofthe acceleration due to gravity g, the tilt angle ofthe solar panel 8, the vibrational intensity w, and the vibrational amplitude A in the equation. Scaling the extrapolated walnut shell durations by a factor of 2.9 aligned them with glass microbead results $(p > 0.05)$, validating chamber rotations as an effective means of gravitational simulation. The power consumed to clean rovers was derived by dividing the power needed to dean our panel by surface area of 5-inch by 7-inch panel and multiplying by the dust removal duration and surface area of the rover's solar panels. Calculations of power required for tilting and vibrating the test panel were scaled up using NASA specifications of rover solar panel areas. Results revealed

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that the In Sight mission (2018) would have allocated approximately 80.8% less ofthe maximum power for dust removal than Spirit/Opportunity (2003), reflecting improvements in panel efficiency over time. As panel efficiency improves in the future, our vibration dust removal will be more valuable to implement, as it requires a lower percentage of maximum power output. Limitations in this study include challenges in achieving 6-7 mbar pressure and vacuum loss during dust deposition. Future work should explore gyroscopic rotation for stability, improve pressure control, and refine dust deposition techniques. Other factors such as rotational behavior ofthe particles, collisions, and air resistance should be accounted for in future calculations.

***K*eywords** Dust mitigation · Solar panel cleaning · Martian environment · Power extrapolation · F orce-balance

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GLOBAL EXISTENCE AND BLOW-UP FOR A NONLINEAR $m(x)-$TRIHARMONIC PARABOLIC EQUATION

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ABSTRACT

In this presentation, we investigate a nonlinear parabolic m(x)-triharmonic parabolic equation with logarithmic source terms. Using the potential well method and the Nehari manifold technique, we investigate the global existence and blow up of solutions.

***K*eywords** Blow up · Global existence · potential well method · variable exponent

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DECAY ESTIMATES FOR A CLASS OF NONLINEAR $m(x)$-TRIHARMONIC PARABOLIC EQUATION

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ABSTRACT

In this work, we investigate the decay estimates for a class of nonlinear $m(x)$-triharmonic parabolic equations with logarithmic source terms. By employing variational techniques and energy methods, we establish the decay of under suitable conditions.

***K*eywords** Decay · global weak solutions · variable exponent

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A Study on Good Practices and International Cooperation in the European Higher Education

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Abstract

In this academic work, an overview of the best practices observed in Higher Education, as well as various relevant and significant European projects, is revisited, reviewed, analyzed, and updated. In addition to the projects developed, promoted, and funded on behalf of the ERASMUS+ Program, managed by the Education, Audiovisual, and Culture Executive Agency (EACEA), the COST Actions, i.e., European cooperation initiatives in science and technology, are also addressed. The last five years' period (2020-2024) is considered here.
The central and main purposes of this study are to reinforce, in a clear and in-depth manner, a comprehensive understanding of the main and most relevant current and emerging trends in education in the field of Operations Research / Management Sciences (OR/MS), with the aim of contributing even more significantly and consistently to a broader, more solid, and deeper knowledge of the various topics related to higher education, including, in particular, relevant, transformative, and impactful contributions in the distinct and important areas of STEM (Science, Technology, Engineering, and Mathematics).
The main and most relevant characteristics associated with good and recommended practices, as well as international cooperation processes in the field of education in OR/MS, are duly addressed and analyzed in two distinct and complementary ways [1]:
A) The main activities in teaching and learning of OR/MS, such as classroom approaches, courses design, courses assessment, immersive/virtual learning, including AI-based activities.
B) The applications of OR/MS tools on Education, such as Big Data Analytics, Data Envelopment Analysis (DEA), Data Mining, Decision Support Systems (DSS), Graph Theory/Combinatorial Optimization, Heuristics and Metaheuristics, Modeling/Simulation, Multi-criteria Decision Analysis (MCDA), Scheduling and/or Timetabling, Routing for school transportation, and Statistics, among others.
The direct and indirect impact of these innovative and structured practices on organizational effectiveness and the overall performance of higher education institutions are discussed and analyzed, so as its amplitude, along with the digital transitions current framework.Additional and complementary developments are also included, either addressing the prospects for treatment of initiatives within the European framework or at international level, including, for example, programs such as the European Social Fund Plus (ESF+) or global reference institutions such as the United Nations Educational, Scientific and Cultural Organization (UNESCO).

Keywords Higher Education · OR/MS tools · COST Actions · ERASMUS+ Projects · Training Actions

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QECCs FROM THE CONSTACYLIC CODES OVER THE RING $R = Fq[v]/\langle v^m - v\rangle$

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ABSTRACT

In their 2024 study titled "Hulls of Constacyclic Codes over Finite Non-Chain Rings and Their Applications in Quantum Codes Construction", Tian, Gao, and Gao investigated $(1-2v)$ −constacyclic codes over the ring $R = \mathbb{F}_q + v\mathbb{F}_q \ \left(v^2 = v\right)$. They presented the generator polynomials of the hulls of such codes and determined their dimensions with respect to both Euclidean and Hermitian inner products. Moreover, under an appropriate Gray map, they proposed two new theorems for constructing quantum error-correcting codes (QECCs) using the X-quantum construction method with respect to Euclidean and Hermitian inner products. As a result, new QECCs with improved parameters compared to existing codes were introduced. This work is extended in the present study by generalizing their results to $\left(-2v^{m-1}+1\right)$ −constacyclic codes over the ring $R = \mathbb{F}_q + v\mathbb{F}_q + \cdots + v^{m-1}\mathbb{F}_q (q = p^s,\, m-1 | \, p-1,\, p$ is prime and $v^m = v)$.This generalization is carried out with respect to the Euclidean inner product.

***K*eywords** Hulls · Quantum Construction X Method · Constacyclic Codes

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A MOVING-SERVER $M/M/1$ QUEUE WITH ZONE-INDUCED INTERRUPTIONS: ERGODIC CONDITION AND CLOSED-FORM STATIONARY DISTRIBUTION

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ABSTRACT

Motivation. Mobile edge platforms—high-speed trains, autonomous vessels, low-altitude drones—serve users only while inside radio-coverage zones; traffic queues up during shadowed stretches. Proper buffer sizing therefore demands a queueing model that couples mobility with intermittent service availability.

Model. Let $\{N_t\}_{t\geq 0}$ be the queue length and $S_t \in \{0,1\}$ indicate whether the server is inside a coverage zone (1) or not (0). Zone lengths are independent exponentials: $\mathrm{Exp}(\alpha)$ for coverage, $\mathrm{Exp}(\beta)$ for out-of-coverage; thus (N_t, S_t) is a continuous-time Markov chain with infinitesimal generator

$$Q = \begin{pmatrix} Q_{11} & Q_{10} \\ Q_{01} & Q_{00} \end{pmatrix}, \qquad Q_{ss'} = \big(q_{ss'}(n,m)\big)_{n,m\geq 0},$$

where $q_{11}(n,n-1) = \mu$, $q_{11}(n,n+1) = \lambda$, $q_{10}(n,n) = \beta$, $q_{01}(n,n) = \alpha$, and all other transition rates are 0. The chain is stochastically equivalent to an $M/M/1$ queue whose server *breaks* with rate α and *repairs* with rate β.

Proposition 1 (Ergodicity). Define $\rho := \lambda(\alpha+\beta)/(\beta\mu)$. The chain is positive recurrent, i.e. the queue is ergodic, *iff*

$$\lambda \;<\; \frac{\beta\mu}{\alpha+\beta} \quad\Longleftrightarrow\quad 0<\rho<1.$$

Proof sketch. A Foster–Lyapunov function $V(n,s) = n + \gamma s$ with suitably chosen $\gamma > 0$ yields $\langle QV,(n,s)\rangle \leq -\varepsilon$ outside a finite set; thus the drift criterion holds.

Proposition 2 (Stationary Distribution). Assuming $0<\rho<1$, the joint stationary probabilities are

$$\pi_{n,1} = \frac{(\lambda+\beta)(1-\rho)}{\lambda+\beta+\alpha}\,\rho^n, \qquad \pi_{n,0} = \frac{\alpha(1-\rho)}{\lambda+\beta+\alpha}\,\rho^n, \quad n\geq 0,$$

so the marginal distribution of N is geometric with parameter $1-\rho$. Expected queue length and waiting time follow in closed form:

$$\mathbb{E}[N] = \frac{\rho}{1-\rho}, \qquad \mathbb{E}[W] = \frac{\rho}{\mu(1-\rho)}.$$

The *physical* parameters—mean coverage length $1/\alpha$, mean shadow length $1/\beta$, and vehicle speed v—enter performance metrics only through α and β. Consequently, increasing coverage density (larger β) or vehicle speed (scaling both rates) widens the stability region linearly. Because probabilities decay geometrically, backlog targets such as $\Pr\{N>k\}\leq\varepsilon$ translate into explicit admissible arrival rates without simulation, enabling real-time buffer sizing for mobile edge platforms.

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CYLINDRICAL REFLECTED ORNSTEIN–UHLENBECK MODEL FOR HIGH-SPEED MOBILE NETWORKS

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ABSTRACT

High-speed trains and motorway platoons form quasi-one-dimensional corridors in which user equipment experiences both rapidly varying path loss and slow correlated shadowing. To capture these phenomena in a unified framework, we introduce a two-dimensional reflected Ornstein–Uhlenbeck diffusion

$$X_t = (Q_t, S_t) \in [q_{\min}, q_{\max}] \times S^1,$$

where S_t is the periodic position of the mobile receiver within a cell and Q_t evolves according to

$$dQ_t = -\alpha\big(Q_t - q(S_t, v_0)\big)\,dt + \sigma\,dW_t + dL_t, \quad dS_t = v_0\,dt \pmod 1,$$

with reflecting boundaries at $q_{\min}$ and $q_{\max}$. The deterministic profile

$$q(s, v_0) = q_0 - a\,d_\varepsilon(s)^\gamma - b\,v_0 - c\ln(1 + v_0)$$

combines a smooth wrap-around path-loss term and a Doppler-induced penalty.

For each fixed speed $v_0 > 0$, this Markov process admits a unique smooth stationary density $\pi_{v_0}(q, s)$ on the cylinder. Up to normalization,

$$\pi_{v_0}(q, s) = \int_0^{1/v_0} \exp\Big\{\tfrac{2\alpha}{\sigma^2}\int_{q_{\min}}^{q}\big(q' - q(s + v_0\tau, v_0)\big)\,dq'\Big\}\,d\tau.$$

This explicit form allows computation of performance metrics without resorting to simulation.

When a single-server queue with Poisson arrivals of rate λ_{in} is coupled to the channel so that the service rate is a function $u(Q_t)$ of the instantaneous signal level, one finds that the joint process of queue length and channel state is positive recurrent if and only if

$$\lambda_{\text{in}} \;<\; \overline{\mu}(v_0) \;=\; \int_{S^1}\int_{q_{\min}}^{q_{\max}} u(q)\,\pi_{v_0}(q, s)\,dq\,ds.$$

This condition provides a clear buffer-dimensioning rule for high-speed scenarios.

By combining wrap-around geometry, reflecting signal bounds, Doppler-aware drift and closed-form stationary analysis, this model furnishes a compact and analytically tractable tool for performance evaluation of mobile aggregators in high-speed rail and highway networks.

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NORM-DEPENDENT OVERSHOOT BOUNDS FOR MULTIDIMENSIONAL RANDOM WALKS WITH APPLICATIONS TO PORTFOLIO RISK

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ABSTRACT

Capital and liquidity buffers must cover not only the first time a cumulative loss process breaches a limit t, but also the *overshoot* R_t beyond that limit. When losses arise from several independent factors, a vector random walk is appropriate; yet classical Lorden bounds are scalar. We obtain explicit, dimension-free bounds for $\mathbb{E}R_t$ under an arbitrary norm, giving stress-test margins for multi-asset portfolios.

Model. Let $Z_1, Z_2, \dots \in \mathbb{R}^d$ be i.i.d. vectors with non-negative components, finite second moment and mean $m = \mathbb{E}Z_1 \neq 0$. Set $S_n = \sum_{k=1}^{n} Z_k$ ($S_0 = 0$). For a threshold $t > 0$ define the first-exit index and overshoot

$$N(t) = \inf\{n \geq 1 : \|S_n\| > t\}, \qquad R_t = \|S_{N(t)}\| - t.$$

Main bound. Fix any norm $\|\cdot\|$ on $\mathbb{R}^d$ and let $C > 0$ satisfy $C\|x\|_1 \leq \|x\|$ for all x. Then, for every $t > 0$,

$$\mathbb{E}R_t \;\leq\; \left(\tfrac{1}{C} + \tfrac{1}{\|\mathbf{1}\|}\right) \frac{\mathbb{E}\|Z_1\|^2 + \mathbb{E}\|Z_1\|_1^2}{\mathbb{E}\|Z_1\|_1}, \qquad \mathbf{1} = (1, \dots, 1)^\top.$$

The right-hand side is independent of the level t and the dimension d.

Idea of proof. Project S_n onto m to obtain a one-dimensional martingale to which the scalar Lorden bound applies; the orthogonal component is then bounded using Cauchy–Schwarz and the constant C. Summing the two contributions yields the stated inequality.

Euclidean-norm corollary. For $\|\cdot\| = \|\cdot\|_2$ one can take $C = d^{-1/2}$ and $\|\mathbf{1}\| = \sqrt{d}$, giving

$$\mathbb{E}R_t \;\leq\; 2\big(\sqrt{d} + d^{-1/2}\big)\, \frac{\mathbb{E}\|Z_1\|_1^2}{\mathbb{E}\|Z_1\|_1}.$$

Risk interpretation. The formula provides a closed-form capital add-on that depends only on first- and second-moment estimates of single-period factor magnitudes. Because neither t nor d appears, the same buffer protects thresholds across all horizons and dimensions, greatly simplifying multi-asset risk management.

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ICMASE 2025

SHARP LORDEN-TYPE BOUNDS FOR THE OVERSHOOT OF MULTIDIMENSIONAL MARTINGALES WITH APPLICATIONS TO RISK ASSESSMENT

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ABSTRACT

Motivation. In risk management a loss process is monitored and a breach is declared when it first exceeds a regulatory threshold b. The main quantity for capital–reserve sizing is the *overshoot* R_b—how far the loss surpasses b. We give moment-based bounds on $\mathbb{E}[R_b]$ for *vector* martingales, delivering dimension-free worst-case estimates for portfolios driven by multiple risk factors.

Model. Let $\{M_n\}_{n\geq 0}$ be an $\mathbb{R}^d$-valued martingale with square-integrable increments $X_n = M_n - M_{n-1}$ and

$$\mu = \mathbb{E}\,\|X_1\| \;>\; 0, \qquad \sigma^2 = \mathrm{Var}\big(\|X_1\|\big).$$

For $b > 0$ define the stopping time and overshoot

$$\tau_b = \inf\{n \geq 1 : \|M_n\| > b\}, \qquad R_b = \|M_{\tau_b}\| - b.$$

Theorem (universal overshoot bound). If $\mathbb{E}[\tau_b] < \infty$, then for *every* $b > 0$

$$\mathbb{E}[R_b] \;\leq\; \frac{\mathbb{E}\,\|X_1\|^2}{\mu}$$

The bound depends only on the first two moments of one increment and is independent of both the dimension d and the level b.

Idea of proof. Set $Y_n = \|M_n\|^2$. Martingale properties yield $\mathbb{E}Y_{\tau_b} = \mathbb{E}\tau_b\,\mathbb{E}\|X_1\|^2$. Jensen gives $\mathbb{E}\|M_{\tau_b}\| \leq \sqrt{\mathbb{E}Y_{\tau_b}}$. Wald's identity $\mathbb{E}\tau_b = (b + \mathbb{E}R_b)/\mu$ then implies the stated inequality.

Risk-buffer corollary. The simpler bound

$$\mathbb{E}[R_b] \;\leq\; \mu + \frac{\sigma^2}{\mu}$$

is often tighter for light-tailed increments and can be interpreted as a conservative capital add-on computable in real time without simulation.

Implications for practice. These moment-only formulas furnish stress-testable limits on average exceedance of solvency or liquidity thresholds by multivariate positions, avoiding dimensional blow-up.

Keywords martingale overshoot · Lorden inequality · risk buffer · stopping time · dimension-free bound

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GEOMETRIC PROPERTIES OF THE WATER AND SALT MOLECULES

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ABSTRACT

This presentation shows the geometric symmetries of two well-known molecules, the water one, specifically snow, and table salt. These two molecules form interesting, but not so complex structures, which are worth analyzing. Most properties are given by the angle at which the chemical bounds are formed, i.e. the angle between the hydrogen molecules and the water is roughly 120o which generates a hexagon-like structure, while the sodium and chlorine bond form, in a larger group, a 90o angle. The dihedral group will be the main tool for the analysis of those respective geometric formations.

Keywords Dihedral, Geometry, Symmetry, Atomic Bonds, Permutations

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INCLUSION PROPERTIES FOR SPIRALLIKE FUNCTION FAMILIES RELATED TO THE ERROR FUNCTION

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ABSTRACT

This paper explores new families of spirallike functions defined through the concepts of subordination and the Error function of the form

$$\mathcal{E}_r f(z) = \frac{\sqrt{\pi z}}{2} er\ f(\sqrt{z}) = z + \Sigma_{n=2}^{\infty} \frac{(-1)^{n-1}}{(2n-1)(n-1)!} z^n.$$

We provide convolution properties, and coefficient bounds for functions within these families and examine their inclusion relations with other well-known classes of analytic functions.

Keywords Analytic function · Spirallike function · Error function · Convolution · Subordination

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