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Operator product expansion for the thermal correlator of scalar currents

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Abstract

In this paper, we have investigated operator product expansion (OPE) for the thermal correlator of the two scalar currents. In extending OPE to finite temperature, some complications arise. One of these complications is the breakdown of Lorentz invariance by the choice of the reference frame. Due to the residual O(3) symmetry, more operators of the same dimension appear in the OPE than those at zero temperature. Taking into account these peculiarities, we calculated Wilson coefficients in the OPE of scalar currents up to mass-dimension four at finite temperature. Obtained results in the $T \to 0$ limit agree with vacuum OPE calculations.

1. Introduction

In order to explain the heavy-ion collision results, some information about hadron parameters at finite temperature and density is required. The investigation of these parameters needs non-perturbative approaches. One of these non-perturbative methods is QCD sum rules [1], formulated by Shifman, Vainshtein and Zakharov. The extension of this method to finite temperature has been made by Bochkarev and Shaposhnikov [2]. In extending these sum rules to finite temperature, two sources of complications arise. One is the interaction of the current with the particles of the medium. This effect requires modifying the dispersion representation. This effect was first included by Bochkarev and Shaposhnikov. They recognized the importance of multiparticle contributions in the spectral function and modified dispersion representation.

The other complication is the breakdown of Lorentz invariance by the choice of the reference frame. Due to the residual O(3) symmetry, more operators of the same dimension appear in the OPE than those at zero temperature [3]. The non-perturbative contribution due to the additional operators was first included in the paper by Hatsuda, Koike and Lee [4]. Taking into account both complications, Mallik first investigated OPE of vector currents using a background field method [5] and applying these results, Mallik and Mukherjee

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obtained the temperature dependence of the ρ-meson parameters [6, 7]. Also, nuclear medium modifications of meson parameters are widely discussed in the literature [8, 9].

In this paper, we have investigated OPE for thermal correlator of the two scalar currents. Throughout this work, we employ the real-time formulation of the thermal field theory [10, 11]. Taking into account finite temperature peculiarities, we calculated Wilson coefficients in the OPE of scalar currents at finite temperature. Obtained results allow us to investigate the temperature dependence of the σ and a_0-mesons parameters.

2. Wilson coefficients

We consider the thermal correlator of the time-ordered product of two scalar currents at finite temperature

$$T(q) = \mathrm{i}\int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}q\cdot x}\langle T(J(x)J(0))\rangle, \tag{1}$$

where $J(x) = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$ is the scalar current in the σ meson channel in QCD theory. The thermal average of an operator A is defined as follows,

$$\langle A\rangle = \mathrm{Tr}\,\mathrm{e}^{-\beta H}\, A/\mathrm{Tr}\,\mathrm{e}^{-\beta H}, \tag{2}$$

where H is the QCD Hamiltonian, and $\beta = 1/T$ stands for the inverse of the temperature. Traces carry out over any complete set of states. Using Wick's theorem and making some transformations, we obtain the following expressions:

$$T(J(x)J(0)) = t_1^{(g)} + t_2^{(q)} + t_3^{(\mathrm{reg})}, \tag{3}$$

where

$$t_1^{(g)} = \mathrm{Tr}\, S(x,0)S(0,x), \tag{4}$$

$$t_2^{(q)} = -\frac{1}{2\pi^2x^4}[mx^2 : \bar{q}q : +2(\not{x})_{ab}x^\mu : \bar{q}_a \mathrm{i}D_\mu q_b :], \tag{5}$$

$$t_3^{(\mathrm{reg})} =: \bar{q}(x)q(x)\bar{q}(0)q(0) : . \tag{6}$$

Here, spinors $q(x)$ describe up or down quarks fields, a and b are spinor indices, and D_μ is the covariant derivative. In our calculations, we used the following representation of $q(x)$ quark fields

$$q(x) = q(0) + x^\mu D_\mu q(0) + \cdots, \tag{7}$$

which takes place in Fock–Schwinger gauge $x^\mu A_\mu^a(x) = 0$. In equation (4), traces carry out over Dirac and color matrices. As seen, at the operators level there is no reference to temperature, but after the calculation of thermal expectation values temperature dependence of the considered quantities appears. The last term in equation (3) is regular and can be neglected. At finite temperature, Lorentz invariance is broken by the choice of a preferred frame of reference. To restore Lorentz invariance in thermal field theory, four-vector velocity of the medium u^μ is introduced ($u^\mu = (1,0,0,0)$ in the rest frame of the medium). Also $S(x,0)$ indicates the propagator for a massless quark field in the background gauge field [5]

$$S(x,0) = \frac{1}{4\pi^2}\left[-\frac{2\not{x}}{x^4} - \frac{\mathrm{i}g}{4x^2}\gamma^\mu\not{x}\gamma^\nu G_{\mu\nu} + \frac{B}{48}(a\not{x} + b\not{u})\right], \tag{8}$$

where $G_{\mu\nu} = \frac{1}{2}\lambda^a G^a_{\mu\nu}$ is the gluon strength tensor, coefficients a and b have the following forms

$$a = 1 - \frac{4}{x^2}(u\cdot x)^2 - 2\ln(-4x^2\mu^2), \tag{9}$$

$$b = 8(u\cdot x)\ln(-4x^2\mu^2). \tag{10}$$

Let us consider the first term in equation (3). Substituting $S(x,0)$ in equation (4) and making some transformations, we meet expressions of kind $\langle \mathrm{tr}^c G_{\alpha\beta}G_{\lambda\sigma}\rangle$. The Lorentz covariance at finite temperature allows us to write

$$\langle \mathrm{tr}^c G_{\alpha\beta}G_{\lambda\sigma}\rangle_T = (g_{\alpha\lambda}g_{\beta\sigma} - g_{\alpha\sigma}g_{\beta\lambda})A - (u_\alpha u_\lambda g_{\beta\sigma} - u_\alpha u_\sigma g_{\beta\lambda} - u_\beta u_\lambda g_{\alpha\sigma} + u_\beta u_\sigma g_{\alpha\lambda})B \tag{11}$$

where $g_{\alpha\beta}$ is the metric tensor, A and B are scalar coefficients. Contracting indices on both sides and using gluonic part of energy–momentum tensor which is defined in the following form

$$\theta^g_{\alpha\beta} = -G^a_{\alpha\lambda}G_\beta{}^{\lambda a} + \tfrac{1}{4}g_{\alpha\beta}G^a_{\lambda\sigma}G^{\lambda\sigma a}, \tag{12}$$

where these coefficients can be expressed with gluon condensate $\langle G^a_{\mu\nu}G^{a\mu\nu}\rangle$ and gluon energy densities $\langle u\theta^g u\rangle$. After some transformations, we obtain the gluonic part of correlation function:

$$\langle t_1^{(g)}\rangle = -\frac{1}{16\pi^4}\left[\frac{16}{x^6} - \frac{aB}{3x^2} - \frac{(u\cdot x)bB}{3x^4} + \frac{g^2}{4x^2}\langle G^a_{\mu\nu}G^{a\mu\nu}\rangle - g^2\frac{z_\beta z^\nu}{2x^4}\langle G^a_{\mu\nu}G^{a\mu\beta}\rangle\right]. \tag{13}$$

Now, we consider the second term in equation (3) for the zero chemical potential case. Expanding the quark bilinear matrix in terms of a complete set of Dirac matrices and taking into account parity conservation, we can write the most general finite temperature decomposition of two operators in the spinor space in the following form [5]:

$$\langle \bar q_a q_b\rangle = C\delta_{ab} + D(\not{u})_{ba}, \tag{14}$$

$$\langle \bar q_a \,\mathrm{i}D_\mu q_b\rangle = U(\gamma_\mu)_{ba} + Vu_\mu(\not{u})_{ba} + Yu_\mu\delta_{ba} + Z(\not{u}\gamma_\mu)_{ba}. \tag{15}$$

Here, C, D, U, V, Y and Z are scalar coefficients. By contracting indices on both sides and using fermionic part of energy–momentum tensor

$$\theta^f_{\mu\nu} = \bar q\gamma_\mu \,\mathrm{i}D_\nu q - \tfrac{1}{4}g_{\mu\nu}m\bar q q, \tag{16}$$

where these coefficients can be expressed with quark condensate $\langle \bar q q\rangle$ and quark energy densities $\langle u\theta^f u\rangle$. Using equalities (14) and (15) for the zero chemical potential case, we get

$$\langle t_2^{(q)}\rangle = -\frac{1}{2\pi^2x^4}[mx^2\langle\bar q q\rangle + 8x^2U + 8(u\cdot x)^2V]. \tag{17}$$

Substituting obtained expressions for coefficients B, U and V in equations (13) and (17), the contributions of operators up to dimension four to correlation function in short distance expansion can be written as

$$\begin{aligned}\langle T(J(x)J(0))\rangle = {} & -\frac{1}{2\pi^2x^4}\left[\frac{3}{2}mx^2\langle\bar q q\rangle - \frac{2}{3}x^2\langle u\theta^f u\rangle + \frac{8}{3}(u\cdot x)^2\langle u\theta^f u\rangle\right] \\ & - \frac{1}{16\pi^4}\left[\frac{16}{x^6} - \frac{aB}{3x^2} - \frac{(u\cdot x)bB}{3x^4} + \frac{g^2}{8x^2}\langle G^a_{\mu\nu}G^{a\mu\nu}\rangle\right. \\ & \left. - \frac{g^2}{6x^2}\langle u\theta^g u\rangle + \frac{2g^2}{3x^4}(u\cdot x)^2\langle u\theta^g u\rangle\right].\end{aligned} \tag{18}$$

Substituting coefficients a and b in equation (18) and applying the Fourier transformation to obtained expression correlation function may be written in the momentum space. As can be seen in the Fourier transformation, we meet different kinds of integrals. For example, one of these integrals has the following form:

$$I = \int \mathrm{d}^4x \, \frac{\ln(-x^2)}{(x^2)^n} (x \cdot u)^2 \, \mathrm{e}^{\mathrm{i}q\cdot x}. \tag{19}$$

Up to now, we have evaluated expressions in the Minkowski space. After that, in order to calculate integrals, we proceed to the Euclidean space and use the following formula

$$\int \mathrm{d}^{4+2\varepsilon} \frac{\mathrm{e}^{-\mathrm{i}Q\cdot x}}{(x^2)^n} = \pi^{2+\varepsilon} \left(\frac{Q^2}{4}\right)^{n-2-\varepsilon} \frac{\Gamma(2+\varepsilon-n)}{\Gamma(n)}, \tag{20}$$

which plays an important role in Fourier transformation calculations, where Q is the Euclidean momentum and $Q^2 = -q^2$. Using the well-known identity

$$\ln x^2 = -\lim_{\varepsilon\to 0} \frac{\mathrm{d}}{\mathrm{d}\varepsilon} \frac{1}{(x^2)^\varepsilon}, \tag{21}$$

the above integral for case $n = 2$ can be transformed to the following form:

$$I = -\mathrm{i}u_\rho u_\beta \lim_{\lambda\to 0} \frac{\mathrm{d}}{\mathrm{d}\lambda} \frac{\partial}{\partial Q_\rho} \frac{\partial}{\partial Q_\beta} \int \mathrm{d}^4x \, \frac{\mathrm{e}^{-\mathrm{i}Q\cdot x}}{(x^2)^{2+\lambda}}. \tag{22}$$

Using the Fourier transformation equality (20) and the $\Gamma(x)$ function expansion formula

$$\Gamma(1+\varepsilon) = 1 - \gamma\varepsilon + O(\varepsilon^2), \tag{23}$$

after some transformations, we have

$$I = \mathrm{i}\pi^2 \left[\left(1 - 2\gamma - \ln\frac{Q^2}{4}\right) A(\omega, Q) + 4\frac{\omega^2}{Q^4}\right], \tag{24}$$

where γ is the Euler constant, $\omega = u \cdot Q$, and $A(\omega, Q)$ has the following form:

$$A(\omega, Q) = \left(\frac{4\omega^2}{Q^4} - \frac{2u^2}{Q^2}\right). \tag{25}$$

After calculating, the other integrals correlation function in the momentum space can be written as

$$\begin{aligned} T(Q) = {} & \frac{3}{Q^2} m\langle\bar{q}q\rangle + \frac{4}{3}\left(A(\omega, Q) - \frac{1}{Q^2}\right)\langle u\theta^f u\rangle + \frac{1}{8\pi^2} Q^2 \left(\gamma - \ln\frac{4\pi}{Q^2}\right) \\ & - \frac{g^2}{72\pi^2}\left[(5+8\gamma)\frac{1}{Q^2} - (1+8\gamma)A(\omega, Q)\right. \\ & \left. - 4\ln\left(\frac{Q^2}{16\mu^2}\right)\left(A - \frac{1}{Q^2}\right) + 16\frac{\omega^2}{Q^4}\right]\langle u\theta^g u\rangle + \frac{g^2}{32\pi^2 Q^2}\left\langle G^a_{\mu\nu} G^{a\mu\nu}\right\rangle, \end{aligned} \tag{26}$$

which reproduces well-known zero temperature result for this correlation function in the $T \to 0$ limit [13]. The large Q^2 behavior of the Wilson coefficients can be improved by the renormalization group equation. Since the operators $m\bar{q}q$, $g^2 G^a_{\mu\nu} G^{a\mu\nu}$ are invariant, their coefficients are also invariant under rescaling process. But the terms including energy–momentum tensors are mixed by the rescaling process. This situation is similar to the deep

inelastic scattering case [12]. Rescaling operators can be expressed as [5]

$$(C_f n_f \theta^f + C_g \theta^g)|_\mu = \frac{3}{16+3n_f} \times \left[\left(n_f \bar{C}_f + \frac{16}{3}\bar{C}_g\right)\Big|_Q \theta + \lambda(Q^2) n_f (\bar{C}_f - \bar{C}_g)|_Q \left(\frac{16}{3}\theta^f - \theta^g\right)\Big|_Q\right] \tag{27}$$

where $\lambda(Q^2) = [\alpha_S(\mu^2)/\alpha_S(Q^2)]^{-\delta/b}$ and $\theta = n_f\theta^f + \theta^g$ are the total energy density of quarks and gluons. To one loop and for n_f quark flavors, the constants δ and b are given by

$$\delta = \frac{2}{3}\left(\frac{16}{3} + n_f\right) \qquad \text{and} \qquad b = 11 - \frac{2}{3}n_f. \tag{28}$$

Note that in the case of mass-dimension four operators *a priori* renormalized at μ are already expressed by the operators evaluated at Q. Only for $\left(\frac{16}{3}\theta^f - \theta^g\right)$does this process of rescaling generate a renormalization group logarithm due to the nonvanishing anomalous dimension δ of this operator. As can be seen from equation (26), the coefficient of θ^g is smaller than θ^f by a factor of g^2. Ignoring including terms g^2 and taking into account equation (27), we finally obtain an improved expression for OPE of scalar currents at finite temperature

$$T(Q) = \frac{1}{8\pi^2}Q^2\left(\gamma - \ln\frac{4\pi}{Q^2}\right) + \frac{3}{Q^2}m\langle\bar{q}q\rangle + \frac{g^2}{32\pi^2Q^2}\left\langle G^a_{\mu\nu}G^{a\mu\nu}\right\rangle + \frac{4}{16+3n_f}\left(A - \frac{1}{Q^2}\right)\left[\langle u\theta u\rangle + \lambda(Q^2)\left(\frac{16}{3}\langle u\theta^f u\rangle - \langle u\theta^g u\rangle\right)\right]. \tag{29}$$

At the end, due to breakdown of Lorentz invariance at finite temperature, two new operators appear in the Wilson expansion, in addition to the two old (Lorentz invariant) ones, already existing in the vacuum sum rules. Therefore, OPE for the thermal correlator is expressed with four temperature-dependent quantities, quark condensate $\langle\bar{q}q\rangle$, gluon condensate $\left\langle G^a_{\mu\nu}G^{a\mu\nu}\right\rangle$, total energy density $\langle u\theta u\rangle$ and the mixed term including quark and gluon energy densities $\left(\frac{16}{3}\langle u\theta^f u\rangle - \langle u\theta^g u\rangle\right)$. All these thermal averages can be evaluated from chiral perturbation theory [14, 15], but an extra input is needed only for the last unphysical operator. Obtained results can be used to determine the temperature dependence of the σ and a_0-meson parameters. It is also essential to investigate OPE including higher dimensions operators. We believe these studies to be of great importance for understanding nonperturbative properties of thermal QCD.

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